1. Invited Talks

Existence results for some nonlinear elliptic problems: are they sharp?

Angelo Alvino, Università degli Studi di Napoli Federico II, Italy

Abstract. Via comparison results we give some optimal a priori bounds for solutions to homogeneous Dirichlet problems of nonlinear elliptic equations whose prototype is

$$-\Delta_p u = |\nabla u|^q + f.$$

Models of electrical conduction in biological tissues

Daniele Andreucci, Sapienza Università di Roma, Italy

Abstract. We review a family of mathematical schemes aimed at modeling the diagnostic device of electric impedance tomography. As a matter of fact, such schemes fit well in a hierarchy comprised also by other well known models of conduction and diffusion in materials with a microstructure, all of which can be investigated by means of homogenization techniques. Essentially, the schemes we have in mind are systems of partial differential equations set in the conductive phases, which are separated by dielectric surfaces, supporting in turn suitable interface conditions.

These models differ obviously from each other in motivations and range of application. However, as far as mathematical analysis is concerned, one can take a unifying point of view by identifying each model by means of the corresponding scaling function relating, roughly speaking, the equations in the bulk with the interface conditions. We follow this lead, reporting, among other issues, on the changes induced in the models by different choices of nonlinear constitutive functions and current frequency ranges.

On the wave equations with $p(x,t)$-Laplacian and with viscous damping: existence and blow-up

Stanislav Antontsev, University of Lisbon, Portugal

Abstract. We study the Dirichlet problem

$$u_{tt} = div\left(a(x,t)\,|\nabla u|^{p(x,t)-2}\,\nabla u + \varepsilon \nabla u_t\right) + b(x,t)|u|^{\sigma(x,t)-2}u,$$
$$(x,t) \in Q_T = \Omega \times (0,T),\ \Omega \in \mathbb{R}^n,$$
$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega,$$
$$u\big|_{\Gamma_T} = 0, \Gamma_T = \partial\Omega \times (0,T).$$

Under suitable condition on the data, we prove local and global existence theorems and study the finite time blow-up of the energy solutions. Also we consider Young measure solutions of this problem with $\varepsilon = 0$.The analysis relies on the methods developed in [1-5].

References

1. Antontsev S.N., Díaz J.I., Shmarev S.I. *Energy Methods for Free Boundary Problems:Applications to Non-linear PDEs and Fluid Mechanics*. Bikhäuser, Boston, 2002. Progress in Nonlinear Differential Equations and Their Applications, Vol. 48.
2. Antontsev S.N. *Wave equation with p(x,t)-Laplacian and damping term: Blow-up of solutions*, C.R. Mecanique, 339, 12(2011),751–755.
3. Antontsev S.N. *Wave equation with p(x,t)- Laplacian and damping term: Existence and blow-up*, Differ. Equ. Appl., 3 (2011), pp. 503–525.
4. Antontsev S.N., Shmarev S. *Blow-up of solutions to parabolic equations with nonstandard growth conditions*. J. Comput. Appl. Math., 234 (2010), pp. 2633–2645.
5. Antontsev S.N., Shmarev S. *Anisotropic parabolic equations with variable nonlinearity*. Publicacions. Sec. Mat. Univ. Autònoma Barcelona, (2009), pp. 355–399.

Numerical Analysis of Time Domain Eddy Current Problems with Intensities or Voltages as Sources

Alfredo Bermúdez, Santiago de Compostela, Spain

Abstract. Joint work with B. López-Rodríguez*, R. Rodrguez* and P. Salgado**

The goal is to analyze a formulation of the time domain eddy current problem in a bounded domain with input current intensities or voltage drops as sources. The chosen formulation is in terms of a time primitive of the electric field. The advantage of this formulation is that no cuts are needed for non-simply connected domains. Firstly, its relation with other formulations used by electrical engineers is addressed. Concerning mathematical analysis, we are led to solve a mixed degenerate parabolic problem involving a Lagrange multiplier associated with the divergence-free gauge condition in the domain of dielectric media. We prove that this continuous problem is well-posed and then we introduce a finite element discretization based on first-order Ndlec approximation spaces for the main unknown and standard nodal finite element of degree one for the Lagrange multiplier. Error estimates are obtained for this semi-discrete problem. Next, the Euler implicit scheme is introduced for time discretization and error estimates are proved for the full discretized problem. Numerical test confirming the expected order of convergence are also included. Finally, the method is applied to compute the eddy currents in a complex geometry which allows us to illustrate the advantage of avoiding cuts in the dielectric domain.

* Universidad de Concepcin. CHILE
** Universidade de Santiago de Compostela. SPAIN

Five Topics in Computational PDEs

Carsten Carstensen, Humboldt-Universität zu Berlin, Germany

Abstract. This presentation is a personal view on the state of the art in five subtopics in computational partial differential equations. The overall goals are reliable error control and efficient simulation and hence the design of optimal algorithm and guaranteed error control in challenging real-life simulations. This supports the general trend to exploit the advantages of nonstandard discretisations in nonlinear CPDEs.

The first topic on the simplest Poisson model problem introduces conforming (CFEM), nonconforming (NC-FEM), and mixed finite element methods (MFEM) and compares them. The errors of CFEM and NC-FEM are equivalent and possibly larger than that of MFEM. This topic is adverised by S. Brenner as an intermediate mathematics between a priori and a posteriori error analysis called medius analysis.

The second topic illustrates the NCFEM for the simplest stationary Stokes equations and presents the quasioptimal convergence with respect to proper approximation classes. There holds equivalence with the pseudo-stress MFEM and optimal convergence of adaptive mesh-refining as well.

Topic three discusses the optimal adaptive computation of elliptic eigenvalue problems and the computation of guaranteed upper and lower eigenvalue bounds based on NCFEM. The particular difficulty in this nonlinear example is the unquanitfied condition of sufficiently small mesh-sizes.

The fourth topic on variational inequalities studies the obstacle problem and its guaranteed error control based on another look due to D. Braess with effectivity indices between 1 and 3. Some novel seemingly quasioptimal adaptive FEM is discussed as well.

Topic five from computational microstructures concerns degenerate minimisation problems and their stabilisation with many open questions on the reliability-efficiency gap. The presentation may end with a first attempt for some MFEM for a topological optimisation problem.

The longest shortest fence and sharp Poincar-Sobolev inequalities

Vincenzo Ferone, Università di Napoli Federico II, Italy

Abstract. The following long standing conjecture concerning the fencing problem in the plane will be addressed: among planar convex sets of given area the disc, and only the disc maximizes the length of the shortest area-bisecting curve. The proof of the above conjecture and some consequences will be presented. Also the fact that among planar convex sets of given area the set which maximizes the length of the shortest bisecting chords is the so-called Auerbach triangle will be discussed.

The 1-harmonic flow with values into a sphere

Lorenzo Giacomelli, Sapienza University of Rome, Italy

Abstract. The 1-harmonic flow is the formal gradient flow of the total variation functional with respect to the L^2-distance. The case in which the unknown is a vector-field, constrained to take values into an hyper-octant of the N-dimensional unit sphere, originates from image processing and has an intrinsic analytical interest as a prototype of constrained and vector-valued evolution equations in BV-spaces. In this lecture I will present an existence result for this problem under homogeneous Neumann boundary conditions, obtained jointly with José Mazon and Salvador Moll. The main difficulty is to give the correct meaning to the Euler-Lagrange equation associated with the 1-energy, since it contains the product between a BV-function and its total variation density. Identifying the jump part of this product requires relaxation-type arguments which lead to a non-trivial and non-convex minimization problem: to find the geodesics in a closed sub-manifold with boundary of the N-sphere, with respect to a metric which penalizes the latitude.

The Mean-Field Limit for a Regularized Vlasov-Maxwell Dynamics

François Golse, Ecole Polytechnique, CMLS, France

Abstract. The Vlasov–Maxwell system is a model equation in the theory of magnetized, collisionless plasmas. The talk proposes a derivation of a regularized variant of this model from a microscopic N particle dynamical system, following earlier results by Neunzert–Wick, Braun–Hepp and Dobrushin in the simpler case of the Vlasov–Poisson system.

Recent advances in Mappings of finite distortion

Stanislav Hencl, Charles University, Czech Republic

Abstract. Mappings of Finite Distortion are mappings from $\mathbb{R}^n \to \mathbb{R}^n$ that may serve as models in Nonlinear Elasticity and in the plane they are closely related with Degenerate Elliptic Equations. We give an overview of some recent results. We study optimal conditions for invertibility and we deal with the problem of orientation preserving. We show that surprisingly it is possible to construct a homeomorphism in the Sobolev space which maps a set of full measure to a null set and the remaining null set to a set of full measure.

Expansions in non-integer bases and a model of a robot's finger

Paola Loreti, Università Sapienza, Italy

Abstract. We will discuss some aspects of the theory of expansions in non-integer bases and a robot finger model in the framework of this theory. The talk is based on a joint work with A. C. Lai.

Research in problem solving: works in progress within obstacles, open questions and some results

Daniela Mansutti, I.A.C.-C.N.R., Rome, Italy

Abstract. I shall introduce several problems directly connected to real applications. After focussing on a particular aspect and a rapid look at the state of art in the related reserch literature, the partial differential model will be described together with its numerical solution procedure. Adopted simplifications will be discussed versus real phenomena characteristics and mathematical difficulties. Numerical techniques or experimental observations are used in order to verify the numerical solutions. Considered study cases are the following: a benchmark for the mathematical numerical modelling of liquid/solid phase change of pure materials, a diagnostic simulation of a subglacial lake in the Svalbard archipelago and an evaluation of plankton surfactants effects on the hydrodynamics of a water basin. Via numerical modelling and simulation within such study cases, we make an attempt to improve the understanding of the impact of differential operators within mathematical modelling of a specific process and to point out necessary improvements, and also to support further experimental investigations.

From Stokes to Darcy in long porous cylinders

Sorin Mardare, University of Rouen, France

Abstract. This work has been done in collaboration with Patrizia Donato from the University of Rouen and Bogdan Vernescu from the Worcester Polytechnic Institute. We study the flow of a fluid in a porous environment represented by a long cylinder containing periodically distributed small obstacles. The applied body forces we are considering are also periodic with the same period as the obstacles. More precisely, we make an asymptotic analysis of the flow of a fluid modeled by the Stokes equations, when the size of the obstacles goes to zero and the length of the cylinder goes to infinity. We combine in this way two already studied phenomena: the homogenization of the Stokes problem and the asymptotic analysis of the same problem in cylinders becoming unbounded in the direction of their axis. We prove that there exists a limit when the two quantities the size of the obstacles and the length of the cylinder - go simultaneously to zero and infinity, respectively. This limit satisfies a Darcy equation in the infinite cylinder. As a consequence of this result, the two limits considered above commute.

Some recent results on optimal Sobolev–Poincaré inequalities

Carlo Nitsch, Università di Napoli Federico II, Italy

Abstract. The first nontrivial Neumann eigenvalue of p-Laplacian provides the best constant in a Sobolev–Poincaré inequality. We prove a sharp lower bound for this eigenvalue and discuss some limiting cases.

Liouville theorems for non-coercive non-cooperative elliptic systems

Pavol Quittner, Comenius University, Slovakia

Abstract. Liouville theorems for scaling invariant problems can be efficiently used in the study of the asymptotic behavior of solutions. We will first consider a model scalar problem in order to explain some consequences of such theorems and then we will present recent results on Liouville theorems for non-coercive non-cooperative elliptic systems of Schrodinger type.

This is a joint work with Philippe Souplet.

Energy identities for conformally invariant variational problems

Tristan Rivière, ETH Zürich, Switzerland

Abstract. Coercive conformally invariant variational problems share common features regarding compactness and regularity. They are called critical in the sense that they sit at the borderline between two categories of problems with opposite behavior : the subcritical ones where regularity and compactness is straightforward and the supercritical ones where regularity and compactness do not hold in general. We will present some aspects of the analysis of critical points to conformally invariant variational problems and we will prove that many aspects of this analysis is coming originally from the study of a general class of linear elliptic systems with antisymmetric potentials. One question we will look more carefully at regards the energy dissipation of sequences of solutions. We will establish in particular a general quantization phenomenon according to which the loss of energy is a sum of well identified quanta.

Correct solvability and spectral analysis of abstract integro-differential equations in a Hilbert space

Victor Vlasov, Russia

Abstract. We obtain the correct solvability of abstract integro-differential equations in a Hilbert space. We study the spectra of the operator-valued functions which are the symbols of above mentioned equations. We analyze the integro-differential equations arising in applications (Gurtin–Pipkin type equations describing the process of heat propagation in media with memory, integro-differential equations arising in the theory of viscoelasticity).

Removable singularities of Lipschitz solutions of fractional Laplace equation

Alexander Volberg, Michigan State University, USA

Abstract. It will be shown that if u is a non-trivial fractional harmonic function in $R^d\mathbb{E}$ with respect to Laplacian to a power b with b strictly between $1/2$ and 1 and it is Lipschitz, then Hausdorff measure of E of dimension $s = d + 1 - 2b$ must be infinite. The result is sharp. We cast it as a fact of non-existence of s-symmetric measures of fractional dimension.

Asymptotic Behavior of Singular Solutions for a Semilinear Parabolic Equation

Eiji Yanagida, Tokyo Institute of Technology, Japan

Abstract. We consider singular solutions for a parabolic partial differential equation with power nonlinearity. It is known that in some range of parameters, this equation has a family of singular steady states with ordered structure. Our concern is the existence of time-dependent singular solutions and their asymptotic behavior. In particular, we prove the convergence of solutions to singular steady states. The method of proofs is based on the analysis of a related linear parabolic equation with a singular coefficient and the comparison principle. This is a joint work with Shota Sato.

2. Short communications & Minisymposia

Nonlinear models for electrical conductions in biological tissues

Micol Amar, Italy

Abstract. We study a hierarchy of electrical conduction problems in biological tissues in the radiofrequency range. These problems are set in a finely mixed periodic medium and the unknown electric potentials solve standard elliptic equations set in different conductive regions (the intracellular and extracellular spaces), separated by an interface (the cell membranes), which exhibits a capacitive and a strongly nonlinear conductive behavior, due to its biochemical structure. As the spatial period of the medium goes to zero, the electric potentials approaches in a suitable sense a homogenization limit u_0, which keeps the prescribed boundary data, and solves a suitable system of equations, depending on the initial scaling. The macroscopic models are obtained by using the technique of two-scale convergence.

On the Existence of Pullback Attractors for non-autonomous parabolic PDEs with dynamical boundary conditions

María Anguiano Moreno, Spain

Abstract. Partial differential equations with dynamical boundary conditions arise for example in hydrodynamics and the heat transfer theory. For instance, they allow to model heat flow inside the considered domain subject to nonlinear heating or cooling at the boundary, or heat transfer in a solid in contact with a moving fluid, in thermoelasticity, heat transfer in two mediums, etc. In this talk we prove the existence and uniqueness of a weak solution for a nonautonomous reaction-diffusion model with dynamical boundary conditions. After that, a continuous dependence result is established via an energy method, including in particular some compactness properties. Finally, the precedent results are used in order to ensure the existence of minimal pullback attractors in the frameworks of universes of fixed bounded sets and that given by a tempered growth condition. The relation among these families is also discussed.

This is a joint work with Prof Pedro Marn-Rubio and Prof José Real.

Fokker Planck-based stochastic optimal control

Mario Annunziato, Italy

Abstract. We formulate an optimal control strategy for stochastic processes by using the Fokker-Planck model equation, with the control objective defined by using the probability density functions. We achive the optimal control as the minimizer of the objective undere the constraint given from the Fokker-Planck equation. We implement a receding-horizon algorithm over time windows in order to attain a final target, or tracking, configuration. Finally, we show examples for 2 dimensional models.

L^∞-estimates for derivatives of solutions to some parabolic free boundary problem

Daria Apushkinskaya, Germany

Abstract. Consider the two-phase parabolic obstacle problem with non-trivial Dirichlet condition

$$\Delta u - \partial_t u = \lambda^+ \chi_{\{u>0\}} - \lambda^- \chi_{\{u<0\}} \quad \text{in } Q = \Omega \times (0;T),$$
$$u = \varphi \quad \text{on} \quad \partial_p Q.$$

Here $T < +\infty$, $\Omega \subset \mathbb{R}^n$ is a given domain, $\partial_P Q$ denotes the parabolic boundary of Q, and $\lambda^\pm$ are non-negative constants satisfying $\lambda^+ + \lambda^- > 0$. The problem arises as limiting case in the model of temperature control through the interior.

In this talk we discuss the L^∞-estimates for the second-order space derivatives $D^2 u$ and the first-order time derivative $\partial_t u$ near the parabolic boundary $\partial_p Q$. Observe that the case of general Dirichlet data cannot be reduced to zero ones due to non-linearity and discontinuity at $u = 0$ of the right-hand side of the first equation.

The talk is based on works in collaboration with Nina Uraltseva.

On Qualitative Properties of Solutions to Quasilinear Ordinary Differential Equations of the Higher Order

Irina Astashova, Russia

Abstract. Uniform estimates will be presented for all solutions with the same domain of the differential nonlinear equation with power nonlinearity:

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x)\, y^{(j)} + p(x)\, |y|^k \operatorname{sgn} y = 0 \tag{1}$$

with $n \geq 1$, $k > 1$, $n, k \in \mathbb{R}$, continuous functions $a_j(x)$ and $p(x)$ such that $|p(x)| \geq p_* > 0$.

Uniform estimates for positive solutions of (1) are obtained in [1].

Uniform estimates for all possible solutions of quasilinear inequalities with different type of nonlinearity

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x)\, y^{(j)} \geq p_*\, |y|^k \tag{2}$$

with $p_* > 0$, $n \geq 1$, $k > 1$, and continuous functions $a_j(x)$ are proved in [2]. In [3] estimates for solutions to this inequality with $a_j(x) \equiv 0$, $j = 0, \ldots, n-1$, are obtained and sufficient conditions for nonexistence of global solutions to this inequality are given.

In [4] uniform estimates are obtained for the absolute values of solutions to this equation with $n = 2$, $a_1(x) \equiv 0$, and the complex-valued potential $p(x)$ whose real part is less than a negative constant.

In [5] for solutions to semi-linear elliptic equations defined in a ball uniform integral estimates were obtained in a smaller closed ball.

The new results about asymptotic behavior of solutions to (1) will be discussed. In particular, we will consider existence of solutions with some special properties. Several earlier results to (1) with $a_j(x) \equiv 0$, $j = 0, \ldots, n-1$, can be found in [6].

These results and methods of proofs can be used in investigations of properties of nonlinear elliptic and parabolic partial differential equations.

The research was supported by RFBR (grant 11-01-00989).

References

[1] Astashova I. V., *Uniform estimates for positive solutions of quasi-linear ordinary differential equations*, Izvestiya: Mathematics, transl. 72 (2008), no 6, 1141–1160.

[2] Astashova I. V., *Uniform estimates of positive solutions to quasi-linear differential inequalities*, Journal of Mathematical Sciences. 143 (2007), no. 4, 3198-3204. Translated from Trudy Seminara imeni I. G. Petrovskogo. 143 (2007), no. 26, 27-36.

[3] Mitidieri E., Pohozhaev S., *A priori Estimates and Blow-up of Solutions to Nonlinear Partial Differential Equations and Inequalities*, Proc. Steklov Inst. Math. 234 (2001), P. 1–383.

[4] Astashova I. V., *Estimates of Solutions to One-dimensional Schrödinger Equation*, World Scientific: Progress in Analysis. Proceedings of the 3rd International ISAAC Congress. Singapore. II (2003), 955–960.

[5] Kondratiev V. A., *On the qualitative properties to solutions of semilinear elliptic equations*, Trudy Seminara imeni I. G. Petrovskogo 16 (1992), 186–190 (Russian). English translation in Proc. of Petrovsky Sem. 16 (1992).

[6] Astashova I. V., *Application of Dynamical Systems to the Study of Asymptotic Properties of Solutions to Nonlinear Higher-Order Differential Equations*, Journal of Mathematical Sciences. Springer Science+Business Media. 126 (2005), no. 5, 1361-1391.

On the first order shape derivative of Kohn–Vogelius cost functional of the Bernoulli problem

Jerico Bacani and Gunther Peichl, Austria

Abstract. In this paper, a free boundary problem of Bernoulli type is being considered. The solution of this Bernoulli problem is studied via shape optimization problem. In particular, the paper focuses on minimizing the Kohn–Vogelius cost functional through the solution of a Pure Dirichlet boundary value problem and a Neumann boundary value problem. The first order shape derivative of the functional is derived without computing the shape derivative of the state variables.

Existence and multiplicity results for a class of problems on unbounded domains

Giuseppina Barletta, Italy

Abstract. We present some existence and multiplicity results for the following problem: Find u in $W^{1,p}(R)$ satisfying $(P_\lambda) - (|u'(x)|^{p-2}u'(x))' + B|u(x)|^{p-2}u(x) = \lambda\alpha(x)g(u(x))$ a.e. in R, where λ is a real positive parameter, B is a real positive number α and g are two given functions.

A mathematical model for alternating pores in biological membranes

Dario Bellaveglia, Italy

Abstract. We consider a mathematical model for selective permeation of chemical species through cell membranes. The mechanism relies on gating, that is on the alternate closing and opening of the pores. Firstly, we study a parabolic problem set in a domain divided by a perforated interface, in the presence of periodic gating. We prove that, for vanishingly small size of the pores and time period, the interface condition prevailing in the limit is a linear relation between the flux (on either side) and the jump of the limiting solution across the interface. Note that such an interface condition only appears when the relative sizes of the relevant geometrical and temporal parameters are suitably connected. In doing so we demonstrate the appearance of a new admissible asymptotic standard with respect to the stationary version of this problem. Secondly, we consider a random walk model based on the same concepts. This study is performed through Monte Carlo numerical techniques, and is aimed at investigating how selective transport, and gating as well, can be obtained by stochastically switching the affinity of the pore for the target species, and at understanding the voltage-dependent changes of ions permeation through cell membranes.

A notion of capacity related to elasticity. Applications to homogenization

Michel Bellieud, France

Abstract. We study a notion of capacity related to elasticity which proves convenient to analyze the concentrations of strain energy caused by the rigid displacements of some infinitesimal parts of an elastic body in two or three dimensions. By way of application, we investigate the behavior of solutions to initial boundary value problems describing vibrations of periodic elastic composites with rapidly varying elastic properties. More specifically, we analyze a two-phase medium whereby a set of heavy stiff tiny particles is embedded in a softer matrix. This task is set in the context of linearized elasticity.

An obstacle problem for capillary surfaces

Josef Bemelmans, Germany

Abstract. Let $G := \Omega \times \mathbb{R}^+, \Omega \subseteq \mathbb{R}^2$ a bounded domain, be a cylinder that is partly filled with liquid; $\mathcal{B}$ is a rigid body that is floating on it, and the interface between the fluid and the air above is described as a graph of a real function u.

If we assume that the shape of the interface is governed by surface tension then the unknowns of the problems, which are the function u and the position of $\mathcal{B}$, are determined by a variational problem for the energy $\mathcal{E}$ of the configuration.

$\mathcal{E}$ consists of the interfacial energy which is proportional to the area of the graph of u, the adhesion energy, which is proportional to the wetted part of the boundary of $\mathcal{B}$ and of the cylindrical boundary $\partial\Omega \times \mathbb{R}^+$ as well as the gravitational energies of the fluid and of the floating body.

Because of the presence of $\mathcal{B}$ the capillary surface Σ is bounded by some curve on $\partial\Omega \times \mathbb{R}^+$ as well as a contact line Γ on $\partial\mathcal{B}$; therefore in our formulation of the variational problem the body $\mathcal{B}$ acts as an obstacle for u.

We show the existence of a minimizer and investigate some of its properties, in particular the regularity. This is based on the first variation of the energy which also gives a variant of Archimedes' principle that includes the forces exerted on $\mathcal{B}$ by Σ.

References

[1] J. Bemelmans, G. P. Galdi, M. Kyed: Fluid Flows around Floating Bodies, I: The Hydrostatic Case, preprint, RWTH Aachen University, to appear in Journal of Mathematical Fluid Mechanics, 2012

[2] J. Bemelmans, G. P. Galdi, M. Kyed: Capillary Surfaces and Floating Bodies, preprint, RWTH Aachen University, 2012

Size estimates for the EIT problem with one measuremnt

Elena Beretta, Italy

Abstract. We estimate the size of a measurable inclusion in terms of power measurements for a single applied boundary current. This problem arises in medical imaging for the screening of organs and one has to deal mathematically with the complex conductivity (admittivity) equation. In this case we are able to establish, for certain classes of admittivities, lower and upper bounds of the measure of the inclusion in terms of the power measurements. Our analysis is based on the derivation of energy bounds and of fine quantitative estimates of unique continuation for solutions to elliptic equations.

Global Lipschitz regularity for the parabolic p-Laplacian system

Verena Bögelein, Germany

Abstract. In this talk we are interested in the boundary regularity of solutions to the parabolic p-Laplacian system. A by now classical result due to DiBenedetto states that the spatial gradients are locally Hölder continuous in the interior. With respect to the boundary regularity the situation is quite different. In the elliptic as well as in the parabolic case it is only known for the equations that solutions are of class $C^{1,\alpha}$ up to the boundary. In this talk we will present a new global Lipschitz regularity result for solutions to the parabolic p-Laplacian system. The result also applies to a larger class of parabolic systems, the so called asymptotically regular systems. The somewhat surprising fact is that no quasi-diagonal structure has to be assumed.

A local minimum theorem and a characterization of the mountain pass geometry

Gabriele Bonanno, Italy

Abstract. The existence of a local minimum for a continuously Gâteaux differentiable function, possibly unbounded from below and without any weak continuity assumption, is established. As a consequence, the existence of one non-zero solution for an elliptic Dirichlet problem with critical exponent is obtained. Moreover, a characterization of the mountain pass geometry is presented. Relations between the mountain pass theorem and local minima are then pointed out and multiplicity results for nonlinear differential problems are established.

On asymptotic analysis of a micropolar fluid flow in thin domain with a free and rough boundary

Mahdi Boukrouche, France

Abstract. We consider a micropolar fluid flow in a 2D domain with a rough and free boundary. We assume that the thickness and the roughness are both of order $0 < \eta << 1$. We prove the existence and uniqueness of a solution of this problem for any value of η and we establish some a priori estimates. Then we use the two-scale convergence technique to derive the limit problem when η tends to zero. Moreover we show that the limit velocity and micro-rotation fields are uniquely determined via auxiliary well-posed problems and the limit pressure is given as the unique solution of a Reynolds equation.

Time-scale atoms chains for image restoration

Vittoria Bruni, Italy

Abstract. In the last years, there has been a growing interest toward signal and image processing methods thanks to the development of digital devices and new technologies and their use in several application fields, like medicine, cultural heritage, geology, biology, nanotechnologies, new materials and so on. The representation of information at different levels of resolution is the key point of the solution to many problems of signal and image processing, including compression, denoising and recognition. In fact, despite their diversity, these problems share the same goal: to identify the significant elements of the signal. Some aspects of the multi-scale wavelet transform will be presented and discussed. Particular attention will be devoted to the study of time-scale evolution of information, establishing both inter and intra relationships between the coefficients of a wavelet decomposition. To this aim the wavelet transform is represented as a linear combination of interfering atoms that correspond to singularity points in the original signal. The time-scale evolution law of such atoms gives the time-scale trajectory of their center of mass. This kind of representation guarantees a stronger compaction as well as a better characterization of the information contained in signals and images. Some applications of these results to compression and denoising of real-world signals and images will be presented.

Waveguide with periodically alternating conditions: homogenization and asymptotics

Renata Bunoiu, France

Abstract. We consider the Laplace operator in a planar infinite strip with frequently periodically alternating Dirichlet and Robin boundary conditions.Assuming that the homogenized boundary condition is the Dirichlet or the Robin one, we establish the uniform resolvent convergence in various operator norms.Moreover, we construct the two-terms asymptotics for the first band functions and the complete asymptotics for the bottom of the spectrum.

Constant-sign and sign-changing solutions for a nonlinear eigenvalue problem

Pasquale Candito, Italy

Abstract. The aim of this talk is to present some recent results, jointly obtained with S. Carl and R. Livrea, concerning the existence of at least three solutions for the following problem:

$$-\Delta_p u = \lambda f(u) \ \text{ in } \ \Omega, \quad u = 0 \ \text{ on } \ \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $p > N$, $-\Delta_p u := -\mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, λ is a positive parameter and f is a continuous function. The approach adopted is based on variational and topological arguments in conjunction with comparison principles and truncation techniques.

First order transmission problems across fractal layers

Raffaela Capitanelli, Italy

Abstract. Transmission problems across irregular interfaces have been used to model many phenomena in nature and technology. The peculiar aspect is that these phenomena take place in domains with small bulk and large interfaces in order to produce rapid and efficient transport. In this talk we deal with the asymptotic homogenization analysis. More precisely, the fractal layer is approximated by a two dimensional insulating thin layer with vanishing thickness and vanishing conductivity. From the PDE's point of view, the limit problem depends on the asymptotic behaviour of ratio between the thickness of the fiber and the conductivity.

Asymptotic analysis of a planar waveguide with small width and "twisted" boundary conditions

Giuseppe Cardone, Italy

Abstract. We consider a planar waveguide with "twisted" boundary conditions. By twisting we mean a special combination of Dirichlet and Neumann boundary conditions. Assuming that the width of the waveguide goes to zero, we identify the effective (limiting) operator as the width of the waveguide tends to zero, establish the uniform resolvent convergence in various possible operator norms, and give the estimates for the rates of convergence. We show that studying the resolvent convergence can be treated as a certain threshold effect and we present an elegant technique which justifies such point of view.

A singular viscoelastic problem: an existence result

Sandra Carillo, Vanda Valente and Giorgio Vergara Caffarelli, Italy

Abstract. A problem of a viscoelastic beam is here considered. Specifically, the case when the viscoelastic relaxation function G(s) exhibits a singularity in the origin is studied. The problem is attacked on introduction of a suitable choice of a sequence of approximating problems. Then, the solution existence follows via compactness method.

Solvability of a generalized Buckley–Leverett model

Nikolai Chemetov, Portugal

Abstract. We propose a new mathematical modeling of the Buckley–Leverett system, which describes the two-phase flows in porous media. We prove the solvability of the initial-boundary value problem for a deduced model, being a coupled system of hyperbolic/elliptic type equations. In order to show the solvability result, we consider an approximated parabolic-elliptic system. Since the approximated solutions do not have ANY type compactness property, the limit transition is justified by the kinetic method. The main issue is to study a linear (kinetic) transport equation, instead of the nonlinear original system.

This result is a joint work with W. Neves (Federal University of Rio de Janeiro).

Localization effect for a spectral problem in a perforated domain

Valeria Chiadò Piat, Italy

Abstract. We consider a homogenization of elliptic spectral problem stated in a perforated domain, Fourier boundary conditions being imposed on the boundary of the perforation. The presence of a locally periodic coefficient in the boundary operator gives rise to a localization effect on the eigenfunctions. Moreover, the limit behaviour of the lower part of the spectrum can be described in terms of an auxiliary harmonic oscillator operator. We describe the asymptotics of the eigenpairs and derive estimates for the rate of convergence.

Torsion in strain-gradient plasticity: energetic scale effects

Maria Chiricotto, Italy

Abstract. We study elasto-plastic torsion in a thin wire in the framework of the strain-gradient plasticity theory elaborated by Gurtin and Anand in 2005. The theory in question envisages two material scales: an energetic scale, which takes into account the so–called "geometrically–necessary dislocations" through a dependence of the free energy on the Burgers tensor, and a dissipative length scale. For the rate-independent case with null dissipative length scale, we construct and characterize a special class of solutions to the evolution problem. With the aid of such characterization, we estimate the dependence on the energetic scale of the ratio between the torque and the twist. Our analysis confirms that the energetic scale is responsible for size–dependent effects, with thinner wires being stronger. We also detect, and quantify in terms of the energetic length scale, both a critical twist, after which the wire becomes fully plastified, and two boundary layers near the externa l boundary of the wire and near the boundary of the plastified region, respectively. This is a joint work with Lorenzo Giacomelli and Giuseppe Tomassetti.

Bounded solutions for a quasilinear singular problem with nonlinear Robin boundary conditions

Imen Chourabi, France

Abstract. This work deals with an existence of bounded positives solutions for a quasilinear singular problem with nonlinear Robin conditions. The nonlinear term is singular with respect to the solution and has a quadratic growth with respect to its gradient. The result is obtained by approximation. Here, the approximate problem does not present the singularity and is bounded with respect to the gradient. In a first step we prove the existence of a bounded solution of this problem. To do that, we apply the Schauder fixed point Theorem. Then, using some equi-integrability arguments we pass to the limit in the approximate problem and obtain at the limit a solution of our problem, we prove also a strong maximum propriety.

Multiple solutions to a magnetic nonlinear Choquard equation

Silvia Cingolani, Italy

Abstract. We consider the stationary nonlinear magnetic Choquard equation where the magnetic vector potential and the scalar potential are compatible with the action of some group G of linear isometries of R^N. We establish the existence of multiple complex valued solutions to this equation.

Multiple solvers approach to simulate water shipping on decks

Giuseppina Colicchio & M. Greco, Italy

Abstract. Water on deck can be a consequence of wave-vessel interactions. It is characterized by compact masses of water entering the ship deck, evolving along it and hitting obstacles on their way. The probability and severity of such a phenomenon is larger for severe sea conditions and incoming wavelengths comparable to the ship length. The consequences depend on the vessel type and its operational conditions. Stability is the main concern for smaller vessels, while structural integrity is of interest for larger ships.

Aiming to achieve a fast, accurate and complete model of this phenomenon, a zonal approach has been devised: a fast Boundary Element Method is used in an outer region modeling the global interaction of the ship with the incoming waves, while a suitable a Navier-Stokes solver is adopted in an inner region very close to the vessel where the water shipping occurs. This phenomenon can be associated with breaking and fragmentation of the air-water interface, vortex shedding and air entrainment, captured by an accurate but computationally expensive NS solver. The challenge in a zonal method is the proper exchange of information between the two solvers across the common boundaries. These involve both a Domain-Decomposition step, where the data from one solver, say donator, are transferred to the other one, say receiver, and a Domain-Composition step, necessary to make the information from the donator consistent with the receiver.

The resulting Domain-Decomposition technique has been validated through the comparison with experiments involving jet run up, water on the deck with entrainment of air cavities, giving good results and making it a promising new tool for designers.

Viscous regularization for Cam-Clay Plasticity

Riccardo Conti, Italy

Abstract.

Impulsive Delay Reaction-Diffusion Cohen–Grossberg Neural Networks with Zero Dirichlet Boundary Conditions

Valery Covachev, Oman

Abstract. An impulsive Cohen–Grossberg neural network with time-varying and S-type distributed delays and reaction-diffusion terms is considered. By using Hardy–Sobolev inequality, under suitable conditions in terms of M-matrices which involve the reaction-diffusion coefficients and the dimension of the spatial domain, it is proved that for the system with zero Dirichlet boundary conditions the equilibrium point is globally exponentially stable. Examples are given.

The exact dependence on $A_2(w)$ for the $L^p(Rz, w)$ maximal inequalities

Luigi D'Onofrio, Italy

Abstract. Buckley proved the linear dependence

$$\|M\|_{L^2(Rz^n,w)} \leq c(n)\, A$$

of the $L^2(Rz^n, w)$– norm for the Hardy-Littlewood maximal operator M on the classical A_2– constant

$$A = A_2(w) = \sup_Q \int_Q w \int_Q w^{-1}$$

where the supremum is taken over all cubes with sides parallel to the axes.

We prove in case $n = 1$, that for $p_0 = 1 + \sqrt{\frac{A-1}{A}} < p \leq 2$, the dependence on the constant A is precisely preserved

$$\|M\|_{L^p(Rz,w)} \leq c(p) \left[\frac{A}{1 - p(2-p)A}\right]^{\frac{1}{p-1}}$$

and it is impossible to decrease the value of p_0.

Similar exact continuation theorems hold for the L^2–norm inequalities of weighted maximal operators.

(Joint work with Roberta Schiattarella)

Radial solutions of entire elliptic equations with mixed powers

Juan Dávila, Chile

Abstract. We consider radial solutions of the elliptic equation in the entire space

$$\Delta u + u^p + u^q - \lambda u = 0, u > 0$$

where $\lambda > 0$ is a parameter.

We prove, with Manuel del Pino (U. de Chile) and Ignacio Guerra (U. de Santiago de Chile), that for some parameter and some subcritical exponents, there are at least 3 solutions. This result settles in a negative way a question about uniqueness of radial solutions to equations of the form $\Delta u + f(u) - u = 0$.

Effective boundary condition for Stokes flow over very rough boundaries

Umberto De Maio, Italy

Abstract. The aim of this talk is to present some results, obtained in a collaboration with Y. Amirat, O. Bodart and A.Gaudiello, about the asymptotic behaviour of viscous fluid flow over very rough boundaries. We consider a viscous incompressible fluid filling an horizontal periodic domain in $\mathbb{R}^3$ delimited at the bottom by a smooth wall and at the top by a rough boundary. We assume that the flow is governed by the stationary Stokes equations. Using asymptotic expansions and boundary layer correctors we construct and analyze an asymptotic approximation of order $\mathcal{O}(\varepsilon^{3/2-\gamma})$ ($\gamma > 0$ being arbitrary small) in the H^1 norm for the velocity, and in the L^2 norm for the pressure. Then we derive an effective boundary condition of Navier type and construct, using the homogenized solution, an asymptotic approximate solution in the whole domain of the flow.

Variational methods for semilinear elliptic equations with measure data

Marco Degiovanni, Italy

Abstract. The talk is devoted to multiplicity results for solutions of semilinear elliptic equations with measure data. They are obtained taking advantage of a direct variational approach, which has been recently developed.

Higher order time asymptotics of fast diffusion by dynamical system approach

Jochen Denzler, USA

Abstract. This presentation will outline higher order asymptotics for the fast diffusion equation, in the relative L^∞ norm. Conflicting requirements on function spaces from the functional analytic and the spectral theoretic side reveal an interplay between convergence rates and tail behavior.

This is joint work with Herbert Koch and Robert McCann.

Numerical simulation of incompressible flows in naval architecture problems

Andrea Di Mascio, Italy

Abstract. The research activity at IAC-CNR on the numerical simulation of flows around ship hulls will be reviewed. Some simulations of seakeeping, manoeuvring and propulsion problems will be shown in order to highlight the main issues related to the simulation of turbulent free surface flows. The focus of the presentation will be on numerical accuracy and uncertainties, space discretization for complex geometries with moving boundaries, parallel computation and turbulence modelling for unsteady flows.

Korteweg–de Vries–Burgers system in R^N

Tomasz Dlotko, Poland

Abstract. Cauchy's problem in R^N for a generalization of the KdV-Burgers system is considered in Sobolev space $(H^{p+1}(R^N))^m$ (here $u = (u_1, ..., u_m)$ is a vector valued function of $x \in R^N$). We want to extend the results obtained recently for the $1 - D$ case [D-S] to the higher space dimension N, concentrating here mainly on the existence and properties of the viscosity solution to the KdV-Burgers system (3). Such higher dimensional problems were studied earlier in [S-R, Z]. Cauchy's problem in R^N we study following that references has the form:

$$\begin{aligned} &u_t + \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \nabla\Phi(u) + \sum_{j=1}^{N} \frac{\partial}{\partial x_j} \sum_{i=1}^{N} \frac{\partial^{2p}}{\partial x_i^{2p}} u = \alpha\Delta u + g(u), \quad t > 0,\ x \in R^N, \\ &u(0, x) = u_0(x),\ x \in R^N. \end{aligned} \tag{3}$$

where $\alpha > 0$, $2p > N \geq 1$ and Φ is a scalar function of the vector $u(t, x) = (u_1(t, x), ..., u_m(t, x))$; ∇ denotes the gradient with respect to u. Global in time solvability and asymptotic behavior of solutions to (3) will be discussed.

References

[D-S] T. Dlotko, Chunyou Sun, *Asymptotic behavior of the generalized Korteweg–de Vries–Burgers equation*, J. Evolution Equations 10 (2010), 571-595.

[S-R] M.E. Schonbek, S.V. Rajopadhye, *Asymptotic behaviour of solutions to the Korteweg–de Vries–Burgers system*, Annales de l'I.H.P., Section C, 12 (1995), 425-457.

[Z] Linghai Zhang, *Decay of solutions of a higher order multidimensional nonlinear Korteweg–de Vries–Burgers system*, Proc. Royal Soc. Edinburgh 124A (1994), 263-271.

On non-monotone approximation schemes for solutions of the second order differential equations

Maria Dobkevich, Latvia

Abstract. The problem (P):$u_{rr} + f(u) = 0, u_r(0) = 0, u(1) = 0$ arises when considering radially symmetric solutions of the problem $\Delta u + \varphi(u) = 0, u = o, x \in \Omega$. The problem (P) is solvable of the upper and lower functions α and β exist. Then there exist solutions u^* and u_* (the maximal and minimal solutions), which can be approximated by monotone sequances if solutions of equation (P). Mostly solutions of (P) are of oscillatory types and they cannot be approximated by monotone sequences. In our talk we provide results on non-monotone approximations of solutions of the poblem (P).

Homogenization of a quasilinear elliptic problem with nonlinear Robin boundary conditions

Patrizia Donato, France

Abstract.
Joint results with Bituin Cabarrubias, University of the Philippines, Diliman.

We present the homogeneization of a quasilinear elliptic equation with oscillating coefficients in a domain periodically perforated by holes of the same size as the period and on the boundary of the holes we prescribe a nonlinear Robin condition, which depend on a real parameter. We suppose that the data satisfy some suitable hypotheses which insure the existence and the uniqueness of a solution of the problem. In particular, suitable growth conditions are assumed on the nonlinear boundary termand on the quasilinear term, some assumptions on the modulus of continuity are prescribed. The physical motivation is that for several composites, like glass or wood, the thermal conductivity depends in a nonlinear way on the temperature itself. On the other hand, nonlinear Robin conditions appear in several physical situations such as climatization or some chemical reactions. We study the convergence to a limit problem, which is identified by using the periodic unfolding method. We also prove the well-posedness of the limit system, showing that the homogenized operator inherits the modulus of continuity of the initial problem. To this aim, we prove a pointwise estimate on the difference of two homogenized matrices for perforated domains in terms of the difference of the original ones. As a consequence, the uniqueness of a solution of the homogenized quasilinear problem follows. The above results are contained in
B. Cabarrubias - P. Donato, *Homogenization of a quasilinear elliptic problem with nonlinear Robin boundary conditions*, Applicable Analysis, (2011), 1-17.

Weak solutions to the heat flow for surfaces of prescribed mean curvature

Frank Duzaar, Germany

Abstract. In this talk we establish the existence of global weak solutions to the heat flow for surfaces of prescribed mean curvature, i.e. the existence for the Cauchy–Dirichlet problem to parabolic systems of the type

$$\begin{cases} \partial_t u - \Delta u = -2(H \circ u) D_1 u \times D_2 u & \text{in } B \times (0,\infty), \\ u = u_o & \text{on } \partial_{\text{par}}\big(B \times (0,\infty)\big), \end{cases}$$

where $H\colon \mathbb{R}^3 \to R$ is a bounded continuous function satisfying an isoperimetric condition, B the unit ball in $\mathbb{R}^2$ and $u\colon B \times (0,\infty) \to \mathbb{R}^3$. As one of the possible applications we show that the problem has a solution with values in $B_R \subset \mathbb{R}^3$, whenever $u_o(B) \subseteq B_R$ and furthermore there holds

$$\int_{\{\xi \in B_R : |H(\xi)| \geq \frac{3}{2R}\}} |H|^3 \, d\xi < \frac{9\pi}{2}, \qquad |H(a)| \leq \tfrac{1}{R} \quad \text{for } a \in \partial B_R.$$

The results that will be presented in the talk are joint work with Verena Bögelein und Chrsitoph Scheven from Erlangen.

Composition Problems Involving Muckenhoupt weights

Fernando Farroni, Italy

Abstract. We consider the composition problem involving Muckenhoupt weights, functions of bounded mean oscillation and quasiconformal mappings. We provide new bounds in terms of some quantities naturally associated to the L^p–regularity of a quasiconformal mappings.

Concentrated Loads on Elastic Unbounded Domains. How and Why Engineers Look(ed) for Green Functions

Antonino Favata, Italy

Abstract. In classical elasticity, there is a class of problems, named after Kelvin, Boussinesq and Mindlin, in all of which a point load induces singular stress fields in unbounded bodies occupying the whole D-dimensional space (with $D = 3$ or 2) or half of it.

In particular, the solution of the Kelvin problem, for which $D = 3$, is the fundamental solution (in modern terminology, the Green function) later used to determine a number of strain nuclei (such as e.g. centers of dilatation and rotation), that is to say, of solutions to problems involving unbounded elastic domains subject to force doublets (or dipoles) with or without moment, in various combinations.

The Kelvin problem is traditionally handled by solving the Navier equations, a system of three coupled elliptic PDEs for the displacement field.

We here propose a different approach, where the basic equations of an elasticity problem – namely, the equilibrium, constitutive, and compatibility equations – are not combined preliminarily into one partial differential equation for displacement, as Navier did, but instead used sequentially, guided by the displacement and stress symmetries intrinsic to the problem at hand.

The optimal convergence of the polynomial collocation method for elliptic pseudodifferential equations

Alexander Fedotov, Russia

Abstract. We prove the convergence of the polynomial collocation method for elliptic pseudodifferential equations in Sobolev spaces via the equivalence between the collocation and modified Galerkin methods. The boundness of the Lagrange interpolation operator in these spaces allows us to obtain the optimal error estimation for the approximate solution, i.e. it has the same rate of convergence as the best approximation of the exact solution by polynomials.

Differential operators on fractal subsets of the real line

Uta Freiberg, Germany

Abstract. Second order differential operators of the form $d/d\mu d/dx$ are introduced on the real line. We discuss analytic properties and give spectral asymptotics for the case that μ is a self similar measure with compact support. Then we extend the results to some more general cases such as random fractal measures and self conformal measures. Moreover, we discuss eigenvalue problems on the real line with Cantor type set boundary and their discrete approximations.

Blow-up Solutions of the Nonlinear Evolution Equations

Sh.N. Gafarova and T.S. Hajiev, Azerbaijan

Abstract. In this paper the unbounded increasing solutions of the nonlinear partial parabolic and hyperbolic equations for the finite time is investigated. The sufficient condition for nonlinearity is established. Under this condition every solution of the investigated problem is blown-up.

Perfect-plasticity for heterogeneous materials

Alessandro Giacomini, Italy

Abstract. Some issues concerning the plastic dissipation along the interface between two phases of a linearly elastic-perfectly plastic composite material will be discussed.

On the local behavior of solutions of logarithmically singular parabolic equations

Ugo Gianazza, Italy

Abstract. The local positivity of solutions to logarithmically singular diffusion equations is investigated in some open space-time domain $E \times (0, T]$. It is shown that if at some time level $t_0 \in (0, T]$ and some point $x_0 \in E$ the solution $u(t_0)$ is not identically zero in a neighborhood of x_0, in a measure-theoretical sense, then it is strictly positive in a neighborhood of x_0. The precise form of this statement is by an intrinsic Harnack-type inequality, which also determines the size of such a neighborhood.

Sharp bounds for composition operator induced by quasiconformal mappings

Raffaella Giova, Italy

Abstract. We provide estimates which involve the composition operator generated by quasiconformal mappings, the distance to L^∞ in some function spaces and the logarithm of the Jacobian of quasiconformal mappings. Our results are sharp in the two dimensional case.

Eddy current losses computation in laminated magnetic cores

Dolores Gómez, Spain

Abstract. This work deals with an axisymmetric transient eddy current model in conductive non-linear magnetic media. The relation between the magnetic field and the magnetic induction, the H-B curve, is non-linear and possibly multivalued (in particular, the slope near the origin can be arbitrarily high). The source is the magnetic flux across a meridian section of the device which leads to a parabolic non-linear problem with non-local boundary conditions. Firstly, we prove existence and uniqueness of solution for a weak formulation written in terms of the magnetic field by applying some abstract results. Then, we compute the numerical solution of the problem by using a finite element method combined with the implicit Euler time discretization. Since the full discrete problem is non-linear and may involve a multi-valued operator, we propose a specific iterative algorithm which is stable with respect to the slope of the H-B curve. Finally, we apply the method to compute the eddy current losses in a toroidal laminated core and show numerical results which asses its performance and in particular, that of the iterative algorithm.

Some new results on linear elliptic PDEs

Luigi Greco, Italy

Abstract. We shall present some new results concerning solvability of Dirichlet problem for second order linear elliptic PDEs and regularity of solutions.

Applications of the Calculus of Moving Surfaces

Pavel Greenfield, USA

Abstract. The calculus of moving surfaces (CMS) is an extension of tensor calculus to deforming manifolds. The central operator in the CMS is the invariant time derivative with respect to a moving surface. The applications of the CMS include physical problems with moving interfaces, boundary variation problems, shape optimization problems, we well as examples, such as a novel proof of the Gauss–Bonnet theorem, where there are no new surfaces. Examples from each class of problems will be presented as well as an in-depth discussion of the novel dynamic fluid film equations.

On Approximation of Higher Order for Some Nonlinear Variational Problems

Joachim Gwinner, Germany

Abstract. In this talk we are concerned with the finite element method in its p-version to treat a scalar variational inequality of the mixed type that models unilateral contact and Coulomb friction or other nonsmooth material behaviour in continuum mechanics. Thus we extend recent work of the author for the boundary element to a larger class of nonlinear nonsmooth variational problems that are treatable by the finite element method.

This leads to a nonconforming discretization scheme. We employ Gauss–Lobatto quadrature for the approximation of the unilateral constraint and also for the friction-type functional and take the resulting quadrature error into account of the error analysis.

At first without any regularity assumptions, we prove convergence of the FEM Galerkin solution in the energy norm. The key of our norm convergence result for the $p-$FEM is the used Gauss–Lobatto integration rule with its high exactness order and its positive weights together with duality arguments in the sense of convex analysis.

Secondly by a novel Céa–Falk lemma we split the total discretization error into two different parts: the distance of the continuous solution to the convex set of approximations in the trial space and the consistency error caused by the nonconforming approximation. Here we use the well-known approximation theory of spectral methods, the cutting technique of Falk, and interpolation arguments. Moreover, we exploit the special structure of the friction-type functional. Thus we arrive under mild regularity assumptions at an a priori error estimate which is suboptimal because of the treatment of the full consistency error in the nonconforming approximation scheme and because of the well-known regularity threshold in unilateral problems.

Vector-valued parabolic quasi-minimizers

Jens Habermann, Germany

Abstract. On a bounded open set $\Omega \subset \mathbb{R}^n$, $n \geq 2$, we consider (possibly vector–valued) parabolic quasi minimizers $u \in L^p(0,T;W^{1,p}(\Omega;\mathbb{R}^N))$, $p > \frac{2n}{n+2}$, related to the Dirichlet p–energy, i.e. functions $u \equiv u(x,t)\colon \Omega \to \mathbb{R}^N$, $N \geq 1$, which satisfy the inequality

$$-\iint_{\operatorname{spt}\varphi} \langle u, \varphi_t \rangle \,\mathrm{d}x\,\mathrm{d}t + \iint_{\operatorname{spt}\varphi} \frac{|Du|^p}{p}\,\mathrm{d}x\,\mathrm{d}t \leq \mathcal{Q} \cdot \iint_{\operatorname{spt}\varphi} \frac{|D(u-\varphi)|^p}{p}\,\mathrm{d}x\,\mathrm{d}t,$$

for all test functions $\varphi \in C_c^\infty(\Omega_T;\mathbb{R}^N)$. Here $\Omega_T := \Omega \times (0,T)$ denotes the parabolic cylincer in $\mathbb{R}^{n+1}$, $\varphi_t \equiv \frac{\partial \varphi}{\partial t}$ the partial time derivative and $\mathcal{Q} \geq 1$ is a constant. We discuss basic regularity questions, such as reverse Hölder type inequalities, higher integrability properties and further regularity results.

Moreover, in a second part of the talk, we give generalizations of the concepts to the situation where $\mathbb{R}^n$ is replaced by a metric measure space $(\mathcal{X}, ,\mu)$ with a doubling measure μ, supporting a Poincaré inequality. In that case, the spatial derivatives Du have to be replaced by (minimal) upper gradients g_u and therefore, the starting point for our discussions are functions $u \in L^p(0,T;N^{1,p}(\Omega;\mathbb{R}^N))$ with the Newtonian space $N^{1,p}(\Omega;\mathbb{R}^N)$. We discuss local higher integrability in the sense that there exists $\varepsilon > 0$ depending only on the data, such that the minimal weak upper gradient g_u locally belongs to $L^{p+\varepsilon}(\Omega)$. Finally, we investigate consequences of this result, concerning further regularity results for quasi– and almost–minimizers.

Certain analytic functions related with elliptic, parabolic and hyperbolic domains

Saqib Hussain, Pakistan

Abstract. In this paper we discuss certain new classes of analytic functions related with conic domains. We derive some inclusion relationships, sharp coefficient bounds and a radius problem. Some other interesting properties of these functions are also investigated.

The weak and strong closures of Sobolev homeomorphisms are the same

Tadeusz Iwaniec, USA

Abstract. Let X and Y be multiply connected Lipschitz domains in the plane. We consider homeomorphisms of X onto Y in the Sobolev class $W^{1,p}(X,Y)$, where $p = 2$ or $p > 2$. Jani Onninen and I proved that the weak and the strong closures are the same thing. This implies, in particular, that the energy-minimal mappings within the class of strong limits of homeomorphisms always exist, a result of considerable interest in mathematical models of nonlinear elasticity.

Global structure of the solution set for a semilinear elliptic problem related to the Liouville equation on an annulus

Toru Kan, Japan

Abstract. We consider the bifurcation structure of a semilinear elliptic problem related to the Liouville equation on a two-dimensional annulus. The problem appears as the limiting problem of the Liouville equation as the inside radius of the annulus tends to 0 and is derived by the method of matched asymptotic expansions. We find explicit solutions including non-radially symmetric ones and determine the connected component containing them. As a consequence, we provide a suggestive evidence for the global structure of the solution set of the Liouville equation.

Asymptotic and numerical analysis for mixed type equation

T.O. Kapustina, J.-P. Lohéac, Russia

Abstract. We consider boundary value problem for elliptic-parabolic equation. We use asymptotic methods and approximated parabolic factorization of elliptic operator in order to construct an efficient numerical algorithm.

Standing Waves for various Schrödinger-Poisson Systems

Otared Kavian, France

Abstract. In this work we report on various results concerning existence of standing wave solutions for the Schrödinger–Poisson system

$$\begin{aligned} -\frac{1}{2}\Delta u + (V+\widetilde{V})u + \omega u &= 0 \\ -\mathrm{div}\left[(1+\varepsilon^4|\nabla V|^2)\nabla V\right] &= |u|^2 - n^*, \end{aligned}$$

either on a periodic domain, or in the whole space $\mathbb{R}^3$. The asymptotic behaviour of the solutions, when the nonlinearity coefficient $\varepsilon > 0$ goes to zero, is also considered. Under appropriate, almost optimal, assumptions on the potential $\widetilde{V}$ and the density n^* we establish existence of a ground state $(u_\varepsilon, V_\varepsilon)$ of the above system, for all ε sufficiently small, and show that $(u_\varepsilon, V_\varepsilon)$ converges to (u_0, V_0), the ground state solution of the corresponding system for $\varepsilon = 0$.

We discuss also the following system

$$\begin{aligned} -\Delta u_j + (V+\widetilde{V})u_j &= \lambda_j u_j \\ -\Delta V &= \sum_{j\geq 1} \alpha_j |u_j|^2 - n^*, \end{aligned}$$

in which $\alpha_j > 0$ and it is moreover required that $\int_\Omega u_j u_k dx = \delta_{jk}$, and the family $(u_j)_{j\geq 1}$ is assumed to be a Hilbert basis of $L^2(\Omega)$.

Mathematics Subject Classification: 35J50, 35Q40. key-words: Schrödinger equation, Poisson equation, standing wave solutions, Variational methods, asymptotic behaviour.

New sixth-order iterative method solving for nonlinear equations

Waseem Asghar Khan, Pakistan

Abstract. In this paper, we suggest and analyze some new higher-order iterative methods by using Householders method free from second derivative for solving nonlinear equations. Per iteration the new methods require three evaluations of the function and two of its first-derivative, so efficiency index equals to 1/5 9 1.552. ; Convergence of their methods are also considered. Several numerical examples are given to illustrate the efficiency and performance of these new methods. These new iterative methods may be viewed as an alternative of the known methods.

A Regularity Criterion for An Axially Symmetric Solution to the Navier-Stokes Equations

Adam Kubica, Poland

Abstract. We consider an axially symmertic solutions of the Navier-Stokes equations. In papers [1] and [2] the authors proved the regularity of solution under the assumption that the radial or angular component of velocity satisfy Serrin-type condition. I will present an analogous result, which holds under weighted version of Serrin-type condition, where the weight is a power of the distance from the axis of symmetry of the solution.

[1] J. Neustupa, M. Pokorny, J. Math. Fluid Mech.2 (2000), no. 4, 381-399.
[2] O. Kreml, M. Pokorny, Elec. J. Diff. Eq., Vol 2007, No. 08, pp.1-10.

Numerical approximation of evolution problems in (pre)fractal domains

Maria Rosaria Lancia, Italy

Abstract. We consider the numerical approximation for a 2D second order parabolic transmission problem across a pre-fractal interface Kn of Koch type, possibily a mixture prefractal set. The layer Kn, a polygonal curve, divides a given domain into two non-convex sub-domains The approximation is carried out by a F.E.M. discretization for the space variable and a finite difference scheme in time. The two main difficulties in the numerical approximation of this type of problems as well as in the simulations are the generation of a suitable mesh to possibly achieve an optimal rate of convergence and to limit the intrinsic computational cost of numeric approximations. We obtain a priori error estimates. By exploiting some regularity results for the solution we build a mesh compliant with the so-called "Grisvard" conditions thus allowing to achieve an optimal rate of convergence both in space and in time.

Joint work with M. Cefalo (Università di Roma, Italy) and H. Liang (Worcester Polytechnic Institue, USA)

Accurate cubature of high-dimensional volume potentials

Flavia Lanzara, Italy

Abstract. This talk is devoted to the efficient computation of high-order cubature formulas for volume potentials obtained within the framework of approximate approximations. We combine this approach with modern methods of structured tensor product approximations. The cubature of the potentials in very high space dimensions is reduced to the quadrature of one-dimensional integrals, leading to a considerable reduction of computing resources.

We present one-dimensional integral represention of high-order cubature formulas for n-dimension harmonic potentials over the whole space, over half-spaces and over rectangular boxes. Numerical experiments confirm the predicted approximation errors. This is a joint work with V. Maz'ya (University of Liverpool and Linköping University) and G. Schmidt (Weierstrass Institute for Applied Analysis and Stochastics, Berlin).

A global existence result for the heat flow of higher dimensional H-systems

Chiara Leone, Italy

Abstract. The talk is concerned with the existence of a global weak solution to the heat flow of the H-system with Cauchy–Dirichlet boundary conditions. The problem is the following:

$$\begin{cases} \partial_t u - \operatorname{div}\left(|\nabla u|^{m-2}\nabla u\right) = & m^{\frac{m}{2}} H(u)\nabla_1 u \wedge \cdots \wedge \nabla_m u \quad \text{in } \Omega_\infty, \\ u = u_0 & \text{on } (\{t=0\}\times\Omega)\bigcup((0,\infty)\times\partial\Omega), \end{cases} \tag{4}$$

for a map $u : \Omega_\infty = (0,\infty)\times\Omega \longrightarrow \mathbb{R}^{m+1}$, Ω bounded open subset of $\mathbb{R}^m$, $H : \mathbb{R}^{m+1} \to \mathbb{R}$ a Lipschitz continuous function and with $u_0 : \Omega \to \mathbb{R}^{m+1}$. We prove the existence of a global weak solution to (4) assuming the smallness (in an appropriate sense) of the initial-boundary datum u_0.

The results presented are based on a collaboration with M. Misawa and A. Verde.

Recent Results on the KPZ Equation

Yuxiang Li, China

Abstract. In this talk, I would like to present the background, the main problems and recent results of KPZ equation: $u_t - \Delta u = |\nabla u|^p$.

Numerical scheme for Laplacian growth models

Jean-Pierre Lohéac, France

Abstract. This work, common with A.S. Demidov (Moscow State University), concerns numerical simulations of Hele–Shaw flows. Our main tool is the Helmholtz–Kirchhoff method. This method allows to transform a free boundary bi-dimensional problem in a fixed boundary problem by introducing a convenient parameterization. It also leads to build numerical schemes. Numerical simulations will be presented and discussed. Especially some numerical experiments show the existence of a critical manifold which can explain some phenomenon of instabilities.

A variable time stepping method for the approximation of retarded potentials

Maria López Fernández, Switzerland

Abstract. We will present a time-stepping method for the approximation of retarded potentials which allows for variable time steps. Our method is a generalization of Lubich's Convolution Quadrature for equidistant time steps. We will introduce the generalized convolution quadrature and develop a theory for its error analysis. This method opens the door for further development towards adaptive time stepping for evolution equations. As the main application of our new theory we will consider the wave equation in exterior domains which are formulated as retarded boundary integral equations.

Multiple solutions to a Dirichlet problem with p-Laplacian and nonlinearity depending on a parameter

Salvatore Angelo Marano, Italy

Abstract. Let Ω be a bounded domain in $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$, let $p \in]1, +\infty[$, and let $j : \Omega \times \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ be a Carathéodory function. Consider the homogeneous Dirichlet problem

$$\begin{cases} -\Delta_p u = j(x, u, \lambda) & \text{in } \Omega\,, \\ u = 0 & \text{on } \partial\Omega\,, \end{cases} \tag{P_λ}$$

where, as usual, $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ for all $u \in W_0^{1,p}(\Omega)$.

In this talk, some multiplicity and bifurcation-type results concerning (P_λ) will be presented by chiefly assuming that either $t \mapsto j(x, t, \lambda)$ exhibits a concave-convex growth rate or $j(x, t, \lambda) := \lambda f(x, t)$ with, roughly speaking,

$$\limsup_{|t|\to+\infty} \frac{f(x,t)}{|t|^{p-2}t} \le 0 < a \le \liminf_{t\to 0} \frac{f(x,t)}{|t|^{p-2}t} < +\infty\,.$$

In both cases, proofs exploit variational methods and truncation techniques; see [1, 2, 3].

References

[1] S. A. Marano and N. S. Papageorgiou, *Positive solutions to a Dirichlet problem with p-Laplacian and concave-convex nonlinearity depending on a parameter,* Comm. Pure Appl. Anal., in press.

[2] S. A. Marano and N. S. Papageorgiou, *Multiple solutions to a Dirichlet problem with p-Laplacian and nonlinearity depending on a parameter,* Adv. Nonlinear Anal., in press.

[3] S. A. Marano, D. Motreanu, and D. Puglisi, *Multiple solutions to a Dirichlet eigenvalue problem with p-Laplacian,* submitted for publication.

Geometric rigidity and the minimization problem in nonlinear elasticity

Cristinel Mardare, France

Abstract. We show that the H1-distance between two deformations of a body is bounded, up to a multiplicative constant, by the L2-distance between their metric tensors. This inequality implies that the total energy of the pure displacement problem in nonlinear elasticity possesses a unique global minimizer for a large class of hyperelastic materials, including that of Saint Venant - Kirchhoff, provided the density of the applied forces are small in Lp-norm.

A variational approach to some quasilinear elliptic variational inequalities

Michele Matzeu, Italy

Abstract. In this talk a semilinear variational inequality with a gradient-dependent nonlinear term is considered. Obviously the nature of this problem is non-variational. Nevertheless it is studied associating a suitable semilinear variational inequality, variational in nature, with it, and performing an iterative technique introduced in [1] in order to treat semilinear elliptic equations when there is a gradient dependence on the nonlinearity.

The existence of a non-trivial non-negative weak solution u for this problem is proved by using essentially variational methods, a penalization technique and an iterative scheme. Via Lewy–Stampacchia's estimates and regularity theory for elliptic equations, the differentiability of u and the α-Hölder continuity of its gradient up to the boundary, for any $\alpha \in (0, 1)$, are shown. Finally, a result on the continuous dependence of this type of solutions on the data, is given.

These results are presented in two joint papers with R. Servadei (see [2, 3]).

References

[1] D. De Figueiredo, M. Girardi and M. Matzeu, *Semilinear elliptic equations with dependence on the gradient via mountain-pass techniques*, Differential and Integral Equations, 17 (1-2) (2004), 119-126.

[2] M. Matzeu and R. Servadei, *Semilinear elliptic variational inequalities with dependence on the gradient via Mountain Pass techniques*, Nonlinear Analysis 72 (2010), 4347-4359.

[3] M. Matzeu and R. Servadei, *Stability for semilinear variational inequalities depending on the gradient*, Nonlinear Analysis 74 (2011), 5161-5170.

On the Homotopy Analysis Method for a mixed problem with a non-local constraint

Said Mesloub, Kingdom of Saudi Arabia

Abstract. By using the Homotopy Analysis Method of Liao, we search for analytic approximate solutions for a mixed problem combining a classical and a non-classical condition (Integral condition). We give some examples where we use the the boundary conditions (partial x-solution) and the initial condition (partial t-solution) to obtain The series solution (approximate and closed form). The HAM allows us to control the convergence region of the obtained series solution.

Variational Analysis in Riemannian and Fractal Geometry

Giovanni Molica Bisci, Italy

Abstract. We present some recent results, obtained in collaboration with G. Bonanno and V. Rădulescu on geometrical variational problems. More precisely, in the first part of the talk, motivated by the Emden-Fowler equation that appears in mathematical physics, we deal with elliptic problems defined on compact Riemannian manifolds without boundary. In the second part, the existence of a sequence of weak solutions for an eigenvalue Dirichlet problem, on a fractal domain, is proved. Our approach is based on variational methods.

On the Dirichlet problem for elliptic equations with lower–order terms

Gioconda Moscariello, Italy

Abstract. We consider the Dirichlet problem for a class of elliptic equations whose model is

$$\mathrm{div}\mathcal{A}(x)\nabla u + E(x)u = \mathrm{div} f.$$

Here $\mathcal{A}(x) = (a_{ij}(x))$ is a symmetric positive definite $N \times N$ matrix and $E(x)$ is a vector field. Under suitable assumptions on $\mathcal{A}(x)$ and $E(x)$, we prove existence and uniqueness results

Hardy–Sobolev–Maz'ya inequalities for the Fractional Laplacian

Luisa Moschini, Italy

Abstract. In this talk we will study trace Hardy and trace Hardy–Sobolev–Maz'ya inequalities with best Hardy constants, for domains satisfying suitable geometric assumptions such as mean convexity or convexity. We will then use them to produce fractional Hardy–Sobolev–Maz'ya inequalities with best Hardy constants for various fractional Laplacians. In the case where the domain is the half space our results cover the full range of the exponent $s \in (0, 1)$ of the fractional Laplacians. In particular we give complete answer in the L^2 setting of an open problem raised by Frank and Seiringer

Anisotropic sigular perturbation method for some alliptic equation of degenerate type

Mohamed Said Moulay, Algeria

Abstract. We study the existence of solutions of some quasi elliptic equation (of second order in some directions and zero in others) of degenerate type. For this purpose we use the anisotropic singular perturbation method. This is joint work with S. Guesmia and R. Kechkar.

On the problem $-\mathrm{div}A(x)Du = f(x) + H(x, Du) + a\,u$ with $|H(x, Du)| \leq C|Du|^2$ and $a > 0$

François Murat, France

Abstract. The existence of a solution for the problem

$$-\mathrm{div}A(x)Du + a_0(x)u = f(x) + H(x, Du) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

and a priori bounds for this solution, have been studied in many papers when $A(x)$ is a coercive matrix with $L^\infty(\Omega)$ coefficients, $a_0(x) \in L^\infty(\Omega)$, $f(x) \in L^{N/2}(\Omega)$, and $H(x, Du)$ is a Caratheodory function which satisfies $|H(x, Du)| \leq C|Du|^2$, in the cases where $a_0(x) \geq \alpha_0 > 0$ and where $a_0(x) = 0$. The solution of this problem has to be searched in the space of functions $u \in H^1_0(\Omega)$ which also satisfy $\exp(\gamma|u|) - 1 \in H^1_0(\Omega)$ for a certain $\gamma > 0$. When $a_0(x) = 0$, smallness conditions have to be imposed on the data.

In this talk I will report on recent joint work with Boussad Hamour for the case where $a_0(x)$ has the bad sign, i.e. satisfies $a_0(x) \leq 0$. We obtain the existence of a (small) solution when smallness conditions are imposed on the data.

One-dimensional linear perfect plasticity as a limit of finite plasticity

Alessandro Musesti, Italy

Abstract. In the framework of the energetic approach to rate independent evolutions, we show that one-dimensional linear perfect plasticity can be obtained by linearization as a variational limit of a finite plasticity model with hardening proposed by A. Mielke in 2004. This is a joint work with Alessandro Giacomini.

Fourth order Brezis-Nirenberg type problems and second-order Caffarelli–Kohn–Nirenberg type inequalities

Roberta Musina, Italy

Abstract. I will discuss some existence-nonexistence phenomena for the following fourth order problem

$$\begin{cases} \Delta^2 u + \lambda \Delta u = |u|^{**-2} u & \text{on } B \\ u = |\nabla u| = 0 & \text{on } \partial B, \end{cases} \tag{5}$$

where B is the unit ball in $\mathbb{R}^n$, $n \geq 5$, $\lambda \in \mathbb{R}$ and $^{**} = 2n/(n-4)$ is the (second order) critical Sobolev exponent.

In particular, I will underline the relevance of (5) with respect to second order dilation-invariant inequalities of the form

$$\int_{\mathbb{R}^n} |x|^{\alpha} |\Delta u|^2 \, dx \geq S_\alpha \left(\int_{\mathbb{R}^n} |x|^{\frac{n\alpha}{n-4}} |u|^{**} \, dx \right)^{2/**} , \quad u \in C_c^\infty(\mathbb{R}^n \setminus \{0\}) .$$

This is a joint paper with Paolo Caldiroli (Milan J. Math (2012)).

On solvability of the critical Neumann problem to elliptic equations with p-Laplacian

Alexander Nazarov, Russia

Abstract. Let Ω be a smooth compact Riemannian n-dimensional manifold with smooth boundary, and let $1 < p < n$. We consider the Neumann problem

$$(I) \qquad -\Delta_p u + u^{p-1} = 0 \quad \text{in}\Omega; \qquad |\nabla u|^{p-2} \frac{\partial u}{\partial \mathbf{n}} = u^{p^\star - 1} \quad \text{on}\partial\Omega$$

(here $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ while $p^\star = (n-1)p/(n-p)$).

We prove that if $\partial\Omega$ contains a point with positive mean curvature (with respect to the interior normal) then the problem (I) has a positive solution for $1 < p < \frac{n+1}{2} + \beta$, where $\beta > 0$ depends on Ω. Further, if the mean curvature of $\partial\Omega$ is non-positive everywhere but $\partial\Omega$ is a totally geodesic submanifold in Ω at some point y^0 with positive scalar curvature then (I) has a positive solution for $2 < p < \frac{n+2}{3} + \gamma$ with $\gamma(\Omega) > 0$. We also consider some related results for more general BVPs.

Similar effects were considered earlier for the critical Dirichlet BVP, see survey [3] and references therein.

This report is partially based on joint papers with Alexander Reznikov [1,2] and supported by RFBR grant 11-01-00825.

References

[1] A.I. Nazarov, A.B. Reznikov, *On the existence of an extremal function in critical Sobolev trace embedding theorem*, J. Funct. Anal. **258** (2010), N11, 3906-3921.

[2] A.I. Nazarov, A.B. Reznikov, *Attainability of infima in the critical Sobolev trace embedding theorem on manifolds*, AMS Transl. Series 2. **229** (2010), 197-210.

[3] A.I. Nazarov, *Dirichlet and Neumann problems to critical Emden–Fowler type equations*, J. Global Optim. **40** (2008), 289–303.

An existence theorem for inextensible nets with slack

Roberto Paroni, Italy

Abstract. Nets formed by two families of fibers that can grow shorter but not longer are studied variationally. It is shown that a minimizing deformation exists and, by means of the dual variational problem, that there exist also tension fibers which equilibrate the applied loads.

Degenerate elliptic equations: existence and regularity results

Antonia Passarelli di Napoli, Italy

Abstract. We illustrate both existence and regularity results for solutions of degenerate elliptic equations, which are contained in the papers [CMP], [MPP], [GGP]. The right hand side is assumed to belong to a suitable Orlicz–Zygmund class. The function that measure the degree of the degeneracy of the equation is exponentially integrable. Some regularity results are also given in case the degeneracy of the equation is subexponentially integrable.

References

[1] G. MOSCARIELLO, A. PASSARELLI DI NAPOLI,M.M. PORZIO, Existence of infinite energy solutions of degenerate elliptic equations, Zeitschrift für Analysis und ihre Anwendungen (ZAA), to appear

[2] F. GIANNETTI, L. GRECO, A. PASSARELLI DI NAPOLI, Regularity of solutions of degenerate A-harmonic equations. Nonlinear Anal. 73, 8 (2010), 2651-2665.

[3] M. CAROZZA, G. MOSCARIELLO, A. PASSARELLI DI NAPOLI, Regularity for p-harmonic equations with right hand side in Orlicz-Zygmund classes. J. Differential Equations 242, 2 (2007), 248-268.

Comparison between Darcy and Brinkman laws in a fracture

Igor Pažanin, Croatia

Abstract. Different laws are used for modeling flows in porous media. Darcy law is the simplest and, by far, the most popular one, due to its simplicity. It states that the filtration velocity of the fluid is proportional to the difference between the body force and the pressure gradient. However, being a first order PDE for the velocity, it cannot sustain the no-slip condition on an impermeable wall or a transmission condition on the contact with free flow. That motivated H.Brinkmann in 1947 to modify the Darcy law. Brinkman model is often used in numerical applications as it allows to resolve problems with boundary conditions on impermeable boundary. In this talk, we derive Darcy and Brinkman law from microscopic equations by upscaling, compare them and estimate the error made by their application.

The obtained result justifies the use of the Brinkman corrector to the Darcy system. This is a joint work with Eduard Marušić-Paloka and Sanja Marušić.

Some singular quasilinear equations with gradient terms

Francesco Petitta, Italy

Abstract. In an open bounded subset of $\mathbb{R}^N$. We will give a description of quasilinear elliptic problems with singular dependence on the unknown in the gradient term. Under suitable assumptions, we describe the sharp range (with respect to the singularity) for existence of positive solutions. Starting from there, we will also discuss possible generalizations for these type of problems involving sign-changing data as well as reaction lower order terms.

On Leray's problem for the stationary Navier-Stokes equations with nonhomogeneous boundary values

Konstantin Pileckas, Lithuania

Abstract. We study the nonhomogeneous boundary value problem for the Navier–Stokes equations

$$\left\{\begin{array}{rcll} -\nu\Delta\mathbf{u}+\big(\mathbf{u}\cdot\nabla\big)\mathbf{u}+\nabla p &=& 0 & \text{in } \Omega,\\ \operatorname{div}\mathbf{u} &=& 0 & \text{in } \Omega,\\ \mathbf{u} &=& \mathbf{a} & \text{on } \partial\Omega \end{array}\right. \tag{1}$$

in a multiply connected domain $\Omega\subset\mathbb{R}^n$ with the boundary $\partial\Omega$, consisting of N disjoint components Γ_j.

Starting from the famous J. Leray's paper published in 1933, problem (1) was a subject of investigation in many papers. The continuity equation in (1) implies the necessary solvability condition

$$\int\limits_{\partial\Omega}\mathbf{a}\cdot\mathbf{n}\,dS=\sum_{j=1}^{N}\int\limits_{\Gamma_j}\mathbf{a}\cdot\mathbf{n}\,dS=\sum_{j=1}^{N}\mathcal{F}_j=0, \tag{2}$$

where $\mathbf{n}$ is a unit vector of the outward (with respect to Ω) normal to $\partial\Omega$. However, for a long time the existence of a weak solution $\mathbf{u}\in W^{1,2}(\Omega)$ to problem (1) was proved only under the stronger condition

$$\mathcal{F}_j=\int\limits_{\Gamma_j}\mathbf{a}\cdot\mathbf{n}\,dS=0,\qquad j=1,2,\dots,N. \tag{3}$$

After J. Leray's paper there were many partial results concerning the sol-vability of problem (1) under the weaker condition (2). In the talk an overview of new results obtained by the author during the last several years will be presented.

Sparsity constraints for the EEG/MEG neuro-imaging inverse problem

Francesca Pitolli, Italy

Abstract. Electroencephalography (EEG) and Magnetoencephalography (MEG) are intended to identify active brain regions through the non-invasive measurements of neuroelectromagnetic signals generated within the brain. EEG and MEG are not true functional neuroimaging techniques since the localization of neural sources requires the reconstruction of the neuroelectric current distribution in the brain once the electromagnetic data are given. Therefore, the localization problem consists in solving an inverse problem that is highly ill-posed and ill-conditioned. As a consequence, the numerical solution of the EEG/MEG inverse problem requires sophisticated numerical schemes and suitable regularization techniques.

In this work we present a fast and efficient numerical scheme for solving the coupled EEG/MEG inverse problem. The scheme uses multiscale methods to compress the discretization matrix associated with the direct problem. Compression allow us to reduce the memory usage and to speed up the numerical solution of the problem. Then, the inverse problem is solved using an iterative thresholding algorithm that acts as a regularization techniques selecting current distributions with a sparse spatial pattern. Finally, an acceleration technique is used to increase the convergence rate.

The numerical tests show that the proposed scheme allows us to localize efficiently neural sources focused in small brain regions.

This work is in collaboration with C. Filardo, M. Muzi, L. Paulon.

Upper bounds for singular perturbation problems

Arkady Poliakovsky, Italy

Abstract. We introduce a new method for constructing upper bounds for general classes of singular perturbation problems. These include problems arising in the study of phase transitions, like the Modica-Mortolla functional, the Aviles-Giga functional and some problems in Micromagnetics.

A bi-phase model of mass diffusion in drug-eluting stents

Giuseppe Pontrelli, Italy

Abstract. In this talk a drug delivery problem of non-equilibrium mass dynamics from a coronary stent to the vascular wall is presented. The drug diffusion in the arterial adjoining porous layers is modeled by a set of coupled PDE's and solved through a spectral decomposition. Drug concentration levels and mass profiles in each layer at various times are given and discussed.

Gradient bounds for elliptic equations singular at the boundary

Alessio Porretta, Italy

Abstract. We present existence and regularity results for a class of elliptic equations in bounded domains with first order terms which are singular at the boundary. Such models arise naturally from viscous Hamilton-Jacobi equations when linearizing around boundary blow-up solutions and are related to stochastic control problems with singular drifts. We discuss the possibility to obtain gradient estimates and the existence of globally Lipschitz solutions provided the transport component of the Hamiltonian is coercive in suitable directions. We also discuss the stability of the estimates in the vanishing viscosity limit and similar results for first order or degenerate problems.

Nonstandard phenomena in some classes of nonlinear elliptic equations

Vicentiu Radulescu, Romania

Abstract. We consider several classes of nonlinear eigenvalue problems with non-homogeneous differential operators. We point out some striking properties, including the existence of a continuous spectrum or concentration phenomena. These results are in relationship with models arising in non-Newtonian "smart" fluids, image restoration or robotics.

Stucture of the solutions of abstract integro-differential equations in a Hilbert space

Nadezda Rautian, Russia

Abstract. We provide the spectral analysis of the operator-functions, which are the symbols of abstract integrodifferential equations in a Hilbert space. We obtain and study the representations of the solutions of these equations as a series of exponentials, using the structure and asymptotic of the spectra of above mentioned equations.

Homogenization of equations describing materials interacting with clouds of particles

Valeria Ricci, Italy

Abstract. We shall describe the derivation of homogenized equations (in an asymptotics of mean-field type) for systems consisting of a cloud of particles dispersed in a material enclosed in a bounded domain. We shall consider in particular the homogenization of the Stokes problem leading to the Brinkman force and the homogenization of a model describing the heat exchange between the material and the dispersed phase leading to a (time-dependent) two-temperature equation. In both cases, the homogenized equations can be rigorously derived using similar form for the correctors. These results were obtained in collaboration with L. Desvillettes and F. Golse.

Minimization for a one-homogenuous functional arising in strain-gradient plasticity

Giuseppe Riey, Italy

Abstract. We consider a minimization problem for a one-homogeneous functional, under the constraint that functions have a prescribed mean value. The problem originates from a one-dimensional strain-gradient theory of plasticity: the minimizer represents the (normalized) time-derivative of the plastic strain in a strip-shaped sample undergoing simple shear with a given shear stress. Of particular interest in this theory is the dependence of solutions on a "dissipative length-scale".

Asymptotic behavior of eigenvalues for problems in cylinders

Prosenjit Roy, Switzerland

Abstract. The talk will be based on the analysis of the asymptotic behavior of the first eigenvalue and eigenfunction for elliptic problems with mixed boundary conditions set on cylinders which tend to become unbounded in some fixed direction. I will show how this problem is connected to a dimension reduction problem.

Facets and regularity of sudden directional diffusion equations

Piotr Rybka, Poland

Abstract. We study two examples. The first one is the total variation flow on an interval with the Dirichlet boundary conditions,

$$u_t - (u_x/|u_x|)_x = 0.$$

We show that facets (i.e. parts of the solutions, where $u_x = 0$) are key elements of the structure of solutions. We present how they are created and they propagate. This is done in relation to regularity of solutions.

The second example is the total variation flow with additional isotropic diffusion,

$$u_t - (u_x/|u_x|)_x - u_{xx} = 0.$$

That is, two different types of diffusions compete here. We show that facets may behave differently.

On upper and lower functions in the second order boundary value problems

Felix Sadyrbaev, Latvia

Abstract. Different types and definitions of upper and lower functions for elliptic boundary value problems are discussed in connection with the existence of monotone and non-monotone approximations to solutions.

On the asymptotic behaviour of some nonlocal boundary value problem with p-Laplace operator

Tetiana Savitska, Switzerland

Abstract. In this talk we shall study the asymptotic behaviour of the solution to a nonlinear parabolic problem of nonlocal type associated to a p-Laplace operator. This leads us also to introduce and investigate the associated stationary problem.

Localizable solutions to nonlinear evolution problems with irregular obstacles: Existence and regularity

Christoph Scheven, Germany

Abstract. We study parabolic obstacle problems related to the evolutionary p-Laplace equation, with a given obstacle function that may be highly irregular and in particular may be increasing in time. For such problems, we introduce a new notion of solution, the main new feature of which is that they solve the problem locally, which is the prerequisite for the derivation of local regularity properties. We present existence results and discuss regularity properties of the constructed solutions.

On the total variations for the inverse of Sobolevand BV homeomorphism

Roberta Schiattarella, Italy

Abstract. Ω be a planar domain. We prove that if f is a homeomorphism of bounded variation, then the inverse f^{-1} has bounded variation and some identities concerning the variations of the components of f^{-1} and the total variation of f hold. As an application, we prove that weak*-compactness in BV holds simultaneously for sequences of BV-homeomorphisms f_j and their inverses f_j^{-1}; this symmetry result fails in the setting of bi-Sobolev mappings.

Kinetic energy estimates and solutions regularity for the 3d Navier-Stokes equations

Vladimir Semenov, Russia

Abstract. In the Cauchy problem for these equations I study the regularity of these solutions and their dependence on the kinetic energy to the time T_0. It gives much better time interval of an existence of the smooth solutions.

Optimization control problems for systems described by elliptic variational inequalities with state constraints

Simon Serovajsky, Kazakhstan

Abstract. The control system described by an elliptic variational inequality is considered. Its solvability is proved. This problem is regularized with using of the penalty method. The convergence of the regularization method is proved. The sequence of solutions for the regularization problem is minimizing sequence for the initial problem. Necessary conditions of optimality for regularization problem are obtained.

Fractional Laplacian equations with subcritical and critical growth

Raffaella Servadei, Italy

Abstract. Motivated by the interest shown in the literature for non-local operators of elliptic type, in some recent papers, joint with Enrico Valdinoci, we have studied problems modeled by

$$\begin{cases} (-\Delta)^s u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega\,, \end{cases} \tag{6}$$

where $s \in (0,1)$ is fixed and $(-\Delta)^s$ is the fractional Laplace operator, $\Omega \subset \mathbb{R}^n$, $n > 2s$, is open, bounded and with Lipschitz boundary, $\lambda > 0$, $2^* = 2n/(n-2s)$ is the fractional critical Sobolev exponent and $2 < q \le 2^*$, that is we considered non-local problems with subcritical ($q < 2^*$) and critical growth ($q = 2^*$). To be precise, in our papers we have studied non-local equations driven by integrodifferential operators more general than the fractional Laplacian.

Problem (6) represents the non-local counterpart of the following nonlinear elliptic equation

$$\begin{cases} -\Delta u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega\,, \end{cases}$$

with $2 < q \le 2_*$, where $2_* = 2n/(n-2)$ and $n > 2$.

Aim of this talk will be to present some results which extend the validity of some existence theorems known in the classical subcritical and critical case of the Laplacian to the non-local framework. In particular, in the critical setting our papers may be seen as the extension of the classical Brezis-Nirenberg result to the case of non-local fractional operators.

The Solution of Initial Boundary Problem for Parabolic and Hyperbolic type Equations by Monte Carlo Methods

Kanat Shakenov, Kazakhstan

Abstract. We consider initial boundary problem for Parabolic and Hyperbolic type equations with the following boundary problems for that's equations: 1) Dirichlet Problem, 2) Neumann Problem and 3) Mixed Problem. After discretization only on time variable all received problems amounts to above-mentioned Problems for equations Elliptic type (total 6 initial boundary Problems) and are solved numerically by means of the algorithms of Monte Carlo methods: "Random walk on spheres", "Random walk on lattices" and Probability Difference Methods.

On uniqueness problems related to the Fokker–Planck–Kolmogorov equation for measures

Stanislav Shaposhnikov, Russia

Abstract. The goal of the talk is to discus the Cauchy problem for Fokker–Planck–Kolmogorov equations with irregular and unbounded coefficients. We give sufficient conditions for uniqueness of probability and integrable solutions and construct examples of nonuniqueness.

On a Class of Doubly Degenerate Parabolic Equations with Variable Exponents of Nonlinearity

Sergey Shmarev, Spain

Abstract. The talk addresses the homogeneous Dirichlet problem for the equation

$$\partial_t \left(|u|^{m(x,t)} \operatorname{sign} u\right) = \sum_{i=1}^{n} D_i \left(a_i(x,t)|D_i u|^{p_i(x,t)-2} D_i u\right) + b(x,t)|u|^{\sigma(x,t)-2}u$$

with given exponents $m(x,t)$, $p_i(x,t)$ and $\sigma(x,t)$. We present new results on the existence of energy solutions in Orlicz-Sobolev spaces with variable exponents and discuss the effects of blow-up and vanishing of solutions in a finite time. Some related results on these issues can be found in [1–4]. The results of the talk were obtained in collaboration with S.Antontsev.

References

[1] S. N. Antontsev and S. I. Shmarev, *Blow-up of solutions to parabolic equations with non-standard growth conditions*, J. Comput. Appl. Math., 234 (2010), pp. 2633–2645

[2] S. N. Antontsev and S. I. Shmarev, *On the blow-up of solutions to anisotropic parabolic equations with variable nonlinearity*, Tr. Mat. Inst. Steklova, 270 (2010), pp. 33–48.

[3] S. N. Antontsev and S. I. Shmarev *Vanishing solutions of anisotropic parabolic equations with variable nonlinearity*, J. Math. Anal. Appl., 361 (2010), pp. 371–391.

[4] S. Antontsev, M. Chipot, and S. Shmarev, *Uniqueness and comparison theorems for solutions of doubly nonlinear parabolic equations with nonstandard growth conditions*, 2012, to appear in Commun. Pure Appl. Anal.

L^p-integrability of the gradient of solutions to quasilinear elliptic systems with discontinuous coefficients

Lubomira Softova, Italy

Abstract. We consider the Dirichlet problem for the following divergence form elliptic system

$$\begin{cases} D_\alpha\big(a_{ij}^{\alpha\beta}(x)D_\beta u^j(x) + a_i^\alpha(x,\mathbf{u})\big) = b_i(x,\mathbf{u},D\mathbf{u}) & \text{a.a. } x\in\Omega \\ \mathbf{u}(x) = 0 & \text{on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^n, n \geq 2$, is Reifenberg flat domain. The operator is supposed to be strongly elliptic with bounded principal coefficients having small BMO norms and the nonlinear terms satisfy controlled growth conditions.

Using the bootstrapping procedure we prove the inclusion $D\mathbf{u} \in L^r(\Omega; M^{n\times N}), r > 2$ for any weak solution $\mathbf{u} \in W_0^{1,2}(\Omega;\mathbb{R}^N) \cap L^\infty(\Omega;\mathbb{R}^N)$ to the above problem.

Analogous result is obtained also for the bounded weak solutions to the Cauchy-Dirichlet problem for divergence form quasilinear parabolic systems with small BMO coefficients.

Harmonic type approximations

Bianca Stroffolini, Italy

Abstract. I will present harmonic-type approximations based on a new Lipschitz truncation and give some applications to problems with general growth.

Asymptotic behavior of elliptic partial differential systems posed in non-periodic rough domains

Francisco Javier Suárez-Grau, Spain

Abstract. The present work is devoted to study the asymptotic behavior of a sequence of elliptic systems posed in a sequence of rough domains Ω_n. The solutions are assumed to belong to a vectorial space $V_n(x)$ depending on $x \in \overline{\Omega}_n$. This permits to consider several types of boundary conditions posed in variables sets of the boundary and in particular contains classical results for the homogenization of Dirichlet elliptic problems in varying domains. Finally, we also show an application to viscous fluids in non-periodic rough domains.

References

[1] Bucur D., Feireisl E., Necasova S., Boundary behavior of viscous fluids: Influence of wall roughness and friction-driven boundary conditions, Archiv. Rational Mech. Anal. 197 (2010) 117-138.

[2] Casado-Diaz J., Luna-Laynez M., Suarez-Grau F.J., The homogenization of elliptic partial differential systems on rough domains with variable boundary conditions, to appear in Proceedings of the Royal Society of Edinburgh, Section: A Mathematics .

[3] Cioranescu D., Murat F., Un term etrange venu dailleurs, Nonlinear partial differential equations and their applications. College de France seminar, Vols II and III. Eds H. Brezis and J.L. Lions. Research Notes in Math. 60 and 70. Pitman, London, 1982, 98-138 and 154-78.

[4] Dal Maso G., Defranceschi A., Vitali E., Integral representation for a class of C1-convex functionals, J. Math. Pures Appl. 9, 73 (1994), 1-46.

Numerical Analysis of a Family of Distributed Elliptic Optimal Control Problems and its Convergence with Respect to a Parameter

Domingo Tarzia, Argentina

Abstract. We consider a bounded domain Ω in $\mathbb{R}^n$ whose regular boundary $\partial\Omega$ consists of the union of two disjoint portions Γ_1 and Γ_2 with $meas(\Gamma_1) > 0$ and $meas(\Gamma_2) > 0$.

The convergence of a family of distributed continuous optimal control problems P_α, governed by elliptic variational equalities, when the parameter α of the family (the heat transfer coefficient on the portion of the boundary Γ_1, i.e. a Robin boundary condition on Γ_1) goes to infinity was studied in Gariboldi-Tarzia, Appl. Math. Optim., 47 (2003), 213-230 (see also Gariboldi-Tarzia, MAT - Serie A, 7 (2004), 31-42). It has been proved that the optimal control, and their corresponding system and adjoint states are strongly convergent, in adequate functional spaces, to the optimal control, and the system and adjoint states respectively of another distributed optimal control problem P, governed also by another elliptic variational equality, with a different boundary condition on the portion of the boundary Γ_1.

We consider the discrete approximations $P_{h\alpha}$ and P_h of the optimal control problems P_α and P respectively, for each $h > 0$ and for each $\alpha > 0$, through the finite element method with Lagrange's triangles of type 1 with parameter h (the longest side of the triangles). We discrete the elliptic variational equalities which define the system and its adjoint state, and the corresponding cost functional of the optimal control problems P_α and P.

The goal of this paper is to study the convergence of this family of distributed discrete elliptic optimal control problems $P_{h\alpha}$ to the distributed discrete elliptic optimal control problem P_h when the parameter α goes to infinity, for each $h > 0$. We prove the convergence of the discrete optimal controls, the discrete system and adjoint states of the family $P_{h\alpha}$ to the corresponding to the discrete optimal control problem P_h when $\alpha \to +\infty$, for each $h > 0$, in adequate functional spaces. Finally, we also obtain a commutative diagram for the optimal control problems $P_{h\alpha}$, P_h, P_α, and P when $\alpha \to +\infty$ and when $h \to +0$ respectively.

Exponential stability for the vibrations modeled by the standard linear model of viscoelastic type

Octavio Paulo Vera Villagran, Chile

Abstract. In this paper we investigate the asymptotic behavior of solutions to the initial boundary value problem for the vibrations modeled by the standard linear model of viscoelastic. Our main result is to establish the exponential stability of the corresponding semigroup.

Semilinear evolution fractal problems

Paola Vernole, Italy

Abstract. We consider a semilinear prabolic transmission problem across either a fractal layer S or the corresponding prefractal layer S_h. We discuss the local existence, uniqueness and regularity of the mild solution in both cases as well as sufficient condition for the global existence. The strong interpretation of the problems will be also studied.

Joint work with M.R. Lancia.

Hypocoercivity: "all-operator" L and the numerics for sedimentation of flocs in SSTs

Tatiana Vierjullie, Chile

Abstract. The very well known theory of hypocoercivity is thought so that it gives a way to consider *all the spatial operators of an evolution equation as the one operator, let us call it "all-operator" L, in space*, thus , considering $\frac{\partial f}{\partial t} + Lf = Q$. In this presentation I will demonstrate why one can not be content with the dimensionally reduced models of floccuated sedimentation in secondary settling tanks, that are a part of wastewater treatment systems. To this end, the theory of hypocoercivity will be applied. The main estimate of the theory, giving the estimate of the rate of convergence to the equilibrium, that is a result of a very complex combination of a dissipative and a conservative operator, will be considered in the sense that for the SSTs, the modeling of its steady state ensures the secure performance of the tank over the days and the adequate prediction of a danger of the overflow of the tank. Morover, as soon as the concept of the "all-operator" L is considered, it is thought very nicely in hypocoercivity that to have the proper estimate for the *combination of the operators* implies that both the dissipative and the conservative operators of the model must be good enough, that is, they should both reflect very good the nature and meet certain requirements of philosophy of the modeling (e.g., reduced operator can not be in a contradiction with its multidimensional counterpart; for a deeper understanding please see [1]). I will prove two theorems (i) it will be demonstrated that conservative operators of dimreduced models of the kind "discontinuous flux" break the domain onto to two subdomains on the independent convergence to the equilibrium, leading to the estimates w.r.t. time that can not be really related with the estimate of the multidimensional model (are erroneously qualitatively different from 2D, 3D estimates) and (ii) the uncloserness of the dimensionally reduced dissipative operator will be proved introducing a new closure for the analysis of sensitivity of the model w.r.t. the chaos in the tank, with the viscosity defined integrally w.r.t. the experiment [2] as

$$\nu_0 = \nu + \frac{C_\mu \, l}{\sigma_{sx} \, L} \int_0^L \sqrt{k} \; dy. \tag{7}$$

The future work will be reported as being motivated by the existence of the divergence form of the source terms of Boltzmann equations as given by the French scientist Fields Medalist C. Villani, thus, I will develop further my finite volume methods for the sedimentation of flocs with possibly integral source terms.

References

[1] Th. Gallay and C. E. Wayne. Long-time asymptotic of the Navier-Stokes equation in R^2 and R^3. Z. Angew. Math. Mech., 86 (2006), pp. 256–267.

[2] D. A. Lyn and W. Rodi. Turbulence measurements in model settling tank. J. Hydraul. Eng., No. 116 (1990), pp. 3–21

Spatial Decay Estimates for Elliptic Integro-Differential Equations

Karen Yeressian Negarchi, Germany

Abstract. For elliptic integro-differential equations, which arise from Levy stochastic processes, we prove spatial decay estimates in the form of weighted norms. Our approach is general, allowing different decay rates. Applications of such estimates include bounding the domain for numerical calculations and proving the existence of solutions in unbounded domains with growing force term at infinity.

Dimensional Reduction for supremal functionals

Elvira Zappale, Italy

Abstract. A 3D-2D dimensional reduction analysis for supremal functionals is performed in the realm of Γ^*-convergence. It is shown also that the supremal representation for the Γ^* limit may be connected, in some case, to the dimensional reduction for integral unbounded functionals.