Book of Abstracts

Swiss-Japanese Seminar
December 17–19, 2012

Reaction diffusion transmission problems

Herbert Amann, Zurich, Switzerland

Abstract. We shall discuss transmission-boundary value problems for parabolic equations in situations where the interface meets the boundary.

Emden Equations in the hyperbolic space

Catherine Bandle, Basel, Switzerland

Abstract. Different types of radial solutions are studied and a fairly complete classification of the solution set will be given. Different methods are used such as linear perturbations, Pohozaev identities and calculus of variations.

This is a common project with Y. Kabeya.

Uniqueness or nonuniqueness...

Bernard Brighi, Université de Haute Alsace, France

Abstract. We will consider the concave solutions of the third order similarity differential equation $f''' + ff'' + g(f') = 0$ and, according to the sign of g, we will discuss the question of uniqueness of the ones that satisfies the boundary conditions $f(0) = a$, $f'(0) = b$ and $f'(t) \to \lambda$ as $t \to +\infty$, where λ is a root of the function g.

On the stability of travelling periodic surface waves with vorticity

Boris Buffoni, Ecole Polytechnique Fédérale - Lausanne, Switzerland

Abstract. With G. R. Burton, we modify the variational method of G. R. Burton and J. F. Toland, so that it becomes suited to a stability analysis.

By stability, we mean conditional energetic stability of the set of minimizers as a whole, the perturbations being spatially periodic of given period.

Analysis of Fractional Harmonic Maps

Francesca Da Lio, ETH Zurich, Switzerland

Abstract. In this talk we present regularity and compactness results for fractional weak harmonic maps into manifolds. These maps are critical points of nonlocal functionals of the form

$$L(u) = \int_{\mathbb{R}^n} |\Delta^{\alpha/2} u(x)|^p dx \,, \tag{1}$$

where $\alpha p = n$, $u \colon \mathbb{R}^n \to \mathcal{N}$, $\mathcal{N}$ is a smooth k-dimensional sub-manifold of $\mathbb{R}^m$ which is compact and without boundary. This kind of variational problems appears as simplified models for *renormalized energy* in general relativity, (see [1]) . For simplicity we consider here the case of weak $1/2-$harmonic maps from the real line into a sphere ($n = 1$, $\alpha = 1/2$, $p = 2$) . In this particular case the Lagrangian (1) is also invariant under the trace of conformal maps that keep invariant the half space $\mathbb{R}^2_+$ (the well-known Möbius group) .

The key point in our results is first a formulation of the Euler-Lagrange equation for (1) (the so-called $1/2-$harmonic map equation) in the form of a nonlocal linear Schrödinger type equation with a *3-terms commutators* in the right-hand-side . We then establish a sharp estimate for these commutators by using the Littlewood-Paley decomposition and the theory of para-products.

References

[1] Alexakis A. & R. Mazzeo The Willmore functional on complete minimal surfaces in H3: boundary regularity and bubbling, arXiv:1204.4955 .

[2] F. Da Lio *Compactness and Bubbles Analysis for Half-Harmonic Maps into Spheres*,arXiv:1210.265 .

[3] F. Da Lio *Fractional Harmonic Maps into Manifolds in odd dimension* $n > 1$,Calc. Var. DOI 10.1007/s00526-012-0556-6 .

[4] F. Da Lio & A.Schikorra *n/p-harmonic maps: regularity for the sphere case*,Adv. Calc.Var. DOI 10.1515/acv-2012-0107 .

[5] F. Da Lio & T. Riviere *3-Commutators Estimates and the Regularity of* $1/2$*-Harmonic Maps into Spheres*,APDE 4-1 (2011), 149–190.

[6] F. Da Lio & T. Riviere *Sub-criticality of non-local Schrödinger systems with antisymmetric potentials and applications to half-harmonic maps*,,Advances in Mathematics 227, (2011), 1300-1348.

[7] A.Schikorra*Regularity of n/2 harmonic maps into spheres* ,Journal of Differential Equations 252 (2012) 1862–1911 .

The pullback equation for symplectic forms

Bernard Dacorogna, Ecole Polytechnique Fédérale - Lausanne, Switzerland

Abstract. An important question in geometry and analysis is to know when two symplectic forms f and g are equivalent. The problem is therefore to find a map φ such that

$$\varphi^* (g) = f.$$

This means that if f and g are two closed $2-$forms with the same rank and are given by

$$g = \sum_{1 \le i < j \le n} g_{ij}(x)\, dx^i \wedge dx^j \quad \text{and} \quad g = \sum_{1 \le i < j \le n} g_{ij}(x)\, dx^i \wedge dx^j$$

then φ should satisfy the system of $n(n-1)/2$ first order partial differential equations given by

$$\sum_{1 \le p < q \le n} g_{pq}(\varphi(x))\, d\varphi^p \wedge d\varphi^q = \sum_{1 \le i < j \le n} f_{ij}(x)\, dx^i \wedge dx^j.$$

We will discuss local and global existence. We will also consider the case where we impose that the map φ is the gradient of a function Φ.

The results have been obtained in collaboration with S. Bandyopadhyay, G. Csato and O. Kneuss and can be found, in part, in the book below.

Csato G., Dacorogna B. et Kneuss O., *The pullback equation for differential forms*, Birkhaüser, PNLDE Series, New York, **83** (2012).

Dissipative solutions of the Euler equations and Onsager's conjecture

Camillo De Lellis, University of Zurich, Switzerland

Abstract. The incompressible Euler equations were derived more than 250 years ago by Euler to describe the motion of an inviscid incompressible fluid. It is known since the pioneering works of Scheffer and Shnirelman that there are nontrivial distributional solutions to these equations which are compactly supported in space and time. If they were to model the motion of a real fluid, we would see it suddenly start moving after staying at rest for a while, without any action by an external force. A celebrated theorem by Nash and Kuiper shows the existence of C^1 isometric embeddings of a fixed flat rectangle in arbitrarily small balls of the three-dimensional space. You should therefore be able to put a fairly large piece of paper in a pocket of your jacket without folding it or crumpling it.

In a first joint work with László Székelyhidi we pointed out that these two counterintuitive facts share many similarities. This has been the starting point of a program to prove a well-known conjecture of Onsager in the theory of turbulence, which states the existence of dissipative Hoelder solutions with any Hoelder exponent smaller than $\frac{1}{3}$. apparent in a more recent result of ours, which proves the existence of continuous solutions which dissipate the kinetic energy. So far we can show the existence of such solutions for exponents smaller than $\frac{1}{5}$.

Large time behaviour of evolution problems in non-cylindrical domains of large size

Senoussi Guesmia, University of Qassim, Saudi Arabia

Abstract. We will deal with a special problem taking into account the asymptotic behaviour of the solutions to evolution problems defined on domains becoming unbounded with respect to the time t. i.e. the state variable domains Ω_ℓ becoming unbounded when t becomes large. (ℓ represents the size of the domain Ω_ℓ). This requires that ℓ depends on t and of course $\ell \to +\infty$ when $t \to +\infty$. It will be shown that the solutions of such problems converge at an exponential rate to some stationary problems.

Continuous and Discrete Helmholtz-type decompositions

Ralf Hiptmair, ETH Zurich, Switzerland

Abstract. The term Helmholtz-type decomposition of $\mathbf{H}(\mathbf{curl},\Omega)$ refers to stable splittings of the form

$$\mathbf{H}(\mathbf{curl},\Omega) = (H^1(\Omega))^3 + \mathbf{grad}\, H^1(\Omega)\ .$$

First mentioned in a work by Birman and Solomyak [1], splittings of this type have quickly become a key tool in both the theoretical and numerical analysis of spaces of **curl**-conforming vectorfields and related variational boundary value problems. They proved instrumental in

- the investigation of extension theorems and trace spaces for $\mathbf{H}(\mathbf{curl},\Omega)$ [2],
- the derivation and regularity and compactness results [3]
- the analysis of boundary integral formulations related to Maxwell's equations [6],
- the design of auxiliary space preconditioners for $\mathbf{H}(\mathbf{curl},\Omega)$-elliptic variational boundary value problems [7].
- the development of a multigrid convergence theory for edge elements [4, 5],

My presentation will start with a proof of the existence of Helmholtz-type decompositions. Then I am going to outline a few of the applications.

References

[1] M. Birman and M. Solomyak, *L_2-theory of the Maxwell operator in arbitrary domains*, Russian Math. Surveys, 42 (1987), pp. 75–96.

[2] A. Buffa, M. Costabel, and D. Sheen, *On traces for $\mathbf{H}(\mathbf{curl},\Omega)$ in Lipschitz domains*, J. Math. Anal. Appl., 276 (2002), pp. 845–867.

[3] M. Costabel, *A coercive bilinear form for Maxwell's equations*, J. Math. Anal. Appl., 157 (1991), pp. 527–541.

[4] R. Hiptmair, *Multigrid method for Maxwell's equations*, SIAM J. Numer. Anal., 36 (1999), pp. 204–225.

[5] ——, *Analysis of multilevel methods for eddy current problems*, Math. Comp., 72 (2003), pp. 1281–1303.

[6] ——, *Coupling of finite elements and boundary elements in electromagnetic scattering*, SIAM J. Numer. Anal., 41 (2003), pp. 919–944.

[7] R. Hiptmair and J. Xu, *Nodal auxiliary space preconditioning in H(curl) and H(div) spaces*, SIAM J. Numer. Anal., 45 (2007), pp. 2483–2509.

Eigenvalue problem for fully nonlinear elliptic operators

Norihisa Ikoma, Tohoku University, Japan

Abstract. In this talk, we consider the eigenvalue problem for fully nonlinear elliptic operators which are radially symmetric when the space dimension is greater or equal to 2. We present the existence result of not only the first eigenvalue but also the higher eigenvalues, namely, the existence of the sequences of eigenvalues. We also prove that there is no eigenvalue other than the ones we found and that each eigenvalue is simple.

This is based on a joint work with Professor Ishii (Waseda university).

On Hardy type inequalities in limiting cases with scale invariance

Norisuke Ioku, Ehime University, Japan

Abstract. We are concerned with Hardy type inequalities in limiting cases with scale invariance. Using the oscillation of rearranged function which is defined by Bennett-Devore-Sharpley, we show Hardy type inequalities in limiting cases with scale invariance. We also show that Hardy's inequality with logarithmic term can be proved as corollary of the result.

Variational problems associated with Trudinger-Moser inequalities in unbounded domains

Michinori Ishiwata, Fukushima University, Japan

Abstract. In this talk, we are concerned with the existence and the nonexistence of maximizers for variational problems associated with the Trudinger-Moser inequality in unbounded domains. Particularly, the nonexistence of maximizers for the subcritical problem is discussed. We also treat the related existence-nonexistence result for higher dimensional case.

Structure-preserving finite difference scheme for the Landau-Lifshitz equation

Tetsuya Ishiwata, Shibaura Institute of Technology, Japan

Abstract. In this talk, we propose a finite difference scheme for the Landau-Lifshitz equation. We show that the scheme inherits length-preserving and energy structures from the original model. We also show that solvability of the scheme since the scheme is implicit and nonlinear and give an error estimate. Finally, we show some numerical results.

Bifurcation diagrams of nonlinear elliptic problems on a spherical cap

Yoshitsugu Kabeya, Osaka Prefecture University, Japan

Abstract. We consider the nonlinear elliptic equation of the form $\Lambda u+\lambda(u^p-u)=0$ on a spherical cap in the unit sphere $\mathbb{S}^n$ with $n \geq 2$ under the homogeneour Dirichlet or Neumann boundary condition, where Λ is the Laplace-Beltrami operator on $\mathbb{S}^n$ and λ is a real parameter. We investigate the bifuraction diagrams to these problems when the spherical cap covers almost all the sphere.

Large time behavior of solutions of a semilinear elliptic equation with a dynamical boundary condition

Tatsuki Kawakami, Osaka Prefecture University, Japan

Abstract. The main purpose of talk is to study the large time behavior of positive solutions of a semilinear elliptic equation with a dynamical boundary condition. We show that small solutions behave asymptotically like suitable multiples of the Poisson kernel.

Standing wavefronts for the FitzHugh-Nagumo system

Yoshihisa Morita, Ryukoku University, Japan

Abstract. We discuss the existence and stability for solutions of planar standing frontwaves in the FitzHugh-Nagumo system. We provide some parameter region to ensure the existence and prove the stability of the solution with symmetric property.

Separation structure of positive radial solutions for semilinear elliptic equations

Yuki Naito, Ehime University, Japan

Abstract. We consider positive radial solutions for semilinear elliptic equations in the whole space. We say that the equation has separation structure if any two positive radial solutions do not intersect each other. In this talk, we will give some remarks on the separation and intersection properties of the solutions.

A resolution of the Yang-Mills-Dirichlet Problem in super-critical dimensions

T. Rivière, ETH Zurich, Switzerland

Abstract. In the early 2000 a series of geometric works, by Donaldson-Thomas and Tian in particular, have stimulated the analysis of Yang-Mills fields in dimension larger than the conformal one. We shall recall the main ingredients, developed mostly in the 80's, for the study of Yang-Mills Fields in critical and sub-critical dimensions. We will explain why most of these ingredients are inoperative in higher dimensions. The purpose of this talk is to present a new alternative framework for the calculus of variations of Yang-Mills Lagrangian in super-critical dimensions. This new framework will permit us to produce in particular minimizing Yang-Mills Fields for arbitrary boundary datas in 5 dimensional manifolds.

Convergence Analysis for Finite Element Discretizations of Highly Indefinite Helmholtz Problems

Stefan Sauter, University of Zurich, Switzerland

Abstract. In this talk, we will consider Galerkin discretizations of highly indefinite Helmholtz problems. Standard a priori and a posteriori error estimates, typically, suffer from the ill-conditioned behaviour of the discrete Galerkin operator in the high-frequency regime. We will prove that - by choosing the polynomial order p in an hp-finite element space according to p=O(log(k)) - the optimal a priori error estimates are preserved. The result is based on a new regularity theory which employs a wave-number dependent frequency splitting of the solution. In this talk, we generalize this regularity theory for the posteriori error estimation of highly indefinite Helmholtz problems.

This talk comprises joint work with Willy Drfler, Markus Melenk, and Asieh Parsania.

Behavior of solutions of some reaction-diffusion equations with autocatalysis property

Kanako Suzuki, Ibaraki University, Japan

Abstract. We consider mathematical models of a pattern formation arising in processes described by a system of a single reaction-diffusion equation coupled with an ordinary differential equation. This type of models exhibits the diffusion-driven instability, and it is expected that non-constant stationary solutions exist and some spatially inhomogeneous solutions converge toward them. However, we see that a certain natural (autocatalysis) property of a system leads to instability of all inhomogeneous stationary solutions.

Since all steady states are unstable, we shall discuss a possible large time behaviour of solutions. We shall show that space inhomogeneous solution of certain models with an autocatalysis property become unbounded in either finite or infinite time, even if space homogeneous solutions are bounded uniformly in time.

This work is a joint work with Anna Marciniak-Czochra (University of Heidelberg), Grzegorz Karch (University of Wroclaw).

Convergence for a 2D elliptic problem with large exponent in nonlinearity

Futoshi Takahashi, Osaka City University, Japan

Abstract. In this talk, we discuss about the asymptotic behavior of solutions to the Dirichlet problem for a two dimensional elliptic equation when its nonlinear exponent gets large. Some concentration phenomena will happen. We establish the precise estimate of the L^∞ norm of solutions and the location of blow up points.

Optimal Korn's inequality for solenoidal vector fields on a periodic slab

Kyoko Tomoeda, Setsunan University, Japan

Abstract. In this talk I will talk about the best constant of Korn's inequality for solenoidal vector fields on a periodic slab. The Korn's inequalities are essential in establishing coercive estimate for boundary value problems of viscous incompressible fluid dynamics. The best constant of Korn's inequality is called Korn constant. Korn constant was investigated for various situations. However in the case of a periodic slab, this is not investigated yet. We obtain Korn constant for solenoidal vector fields on a periodic slab. The proof of this result was based on Ito's works.

Exact solutions and applications to reaction diffusion systems

Tohru Wakasa, Kyushu Institute of Technology, Japan

Abstract. In this talk we are concerned with a reaction diffusion system of bistable type. Some exact solutions to the system are given and are applied to understand dynamical theory for the system.

Spreading and vanishing dichotomy for some free boundary problems in ecology

Yoshio Yamada, Waseda University, Japan

Abstract. This talk is concerned with free boundary problems which model the invasion or migration of a species. The population density of the species is described by a reaction-diffusion equation and a part of the boundary of its habitant is a free boundary, whose dynamics is determined by a Stefan-like condition. Our purpose is to study asymptotic behavior of solutions as time goes to infinity. In this direction, the spreading of species is the case when the free boundary spreads out and the population density remains positive everywhere as time goes to infinity. On the other hand, the vanishing is the case when the species becomes extinct eventually. We will establish a spreading and dichotomy theorem for suitable classes of reaction diffusion equations. Furthermore, we also give some sufficient conditions for spreading and/or vanishing of species.

Non-accessible singular homoclinic orbits for a semilinear parabolic equation

Eiji Yanagida, Tokyo Institute of Technology, Japan

Abstract. We show the existence of at least three different continuations beyond blow-up for a backward self-similar solution of a supercritical Fujita equation. One of these extended solutions cannot be approximated by classical entire solutions in a specific way given by the scaling invariance of the equation, while the minimal continuation is known to be accessible by a family of such entire solutions. This is a joint work with Marek Fila.