1. Invited Talks

PDE methods in machine learning

Andrea Bertozzi, University of California Los Angeles, USA

Abstract. Geometric methods based on PDEs have revolutionized the field of image processing and image analysis. I will discuss recent work to develop these ideas for machine learning applications involving "big data". The main idea is to pose variational problems involving graph cuts in terms of total variation minimization problems. We then develop both phase field and mean curvature methods to solve these problems quickly. I will introduce the notion of the Ginzburg-Landau functional on graphs and the related dynamic thresholding method. Unlike numerical methods for PDEs, the graph problems are able to exploit dramatic spectral truncation of the graph Laplacian, sometimes with a tiny fraction of the eigenfunctions. I will show examples from semi-supervised learning, nonlocal means image processing, and modularity optimization for unsupervised learning and community detection in networks.

Stability and Pattern formation in Nonlocal Interaction Models

José Antonio Carrillo, Imperial College London, Great Britain

Abstract. I will review some recent results for first and second order models of swarming in terms of patterns, stationary states, and qualitative properties. I will discuss the stability of these patterns for the continuum and discrete particle cases.

These non-local models appear in collective behavior for animals, control engineering, and molecular structures among others. We first concentrate in the spatial shape of these patterns and the dynamics when inertia terms are neglected. The mathematical question behind consists in finding properties about local minimizers of the total interaction energy. Concerning 2nd order models, we will discuss particular properties of two patterns: flocks and mills. We will discuss the stability of these patterns in the discrete case. In both cases, we will describe the properties obtained for the continuum limits.

Gradients of j-elliptic energies and applications

Ralph Chill, TU Dresden, Germany

Abstract. We review the classical theory of subgradients and subgradient systems of convex and elliptic energies on Hilbert spaces. The settings which can be found, for example, in the classical books by J.-L. Lions (Quelques methodes de resolution des problemes aux limites non-lineaires) and Brezis (Operateurs maximaux monotones et semi-groupes de contractions dans les espaces de Hilbert) are presented in a unified and extended way. Possible applications include the Dirichlet-to-Neumann operator and nonlinear variants, as considered recently by Arendt and ter Elst, or Hauer and Kennedy.

On the pullback equation for differential forms

Bernard Dacorogna, EPFL, Switzerland

Abstract. Given two functions f and g, we want to find a map φ such that

$$\begin{cases} g(\varphi(x)) \det \nabla\varphi(x) = f(x) & x \in \Omega \\ \varphi(x) = x & x \in \partial\Omega. \end{cases}$$

(i) We first discuss the case where $g \cdot f > 0$ and give three different ideas for the existence problem with optimal regularity.

(ii) We then briefly comment on the case where $g > 0$ but f is allowed to change sign.

(iii) We finally consider the (local) existence, uniqueness and optimal regularity for the problem

$$g_i(\varphi(x)) \det \nabla\varphi(x) = f_i(x) \quad \text{for every } 1 \leq i \leq n$$

where $g_i \cdot f_i > 0$.

Construction of nonlocal minimal surfaces

Juan Davila, Universidad de Chile, Chile

Abstract. We construct the first non trivial examples of nonlocal minimal surfaces. These surfaces are stationary for a notion of nonlocal perimeter introduced by Caffarelli, Roquejoffre and Savin. We also discuss stability properties of the Lawson cones in this context.

Critical group estimates for functionals associated with quasilinear elliptic equations

Marco Degiovanni, Catholic University of the Sacred Hearth, Italy

Abstract. We consider quasilinear elliptic equations that include p-Laplace equations for $1 < p < 2$. We provide some estimates $\hat{A}$ for critical groups of the associated functional.

Motility at microscopic scales

Antonio DeSimone, SISSA, Italy

Abstract. Motility of cells is at the root of many fundamental processes in biology: from sperm cells swimming to fertilize an egg cell, to metastatic tumor cells crawling to invade nearby tissues.

We will discuss the mechanical bases of cellular motility by swimming and crawling. Special emphasis will be placed on the connections between low Reynolds number swimming and Geometric Control Theory, and on the geometric structure of the underlying equations of motion.

As a concrete example, we will report on reverse engineering of the euglenoid movement. The lessons learned in the context of swimming motility will be then applied to selected case studies of crawling motility.

Variational Methods for Crystal Surface Instability

Irene Fonseca, Carnegie Mellon University, USA

Abstract. Using the calculus of variations it is shown that important qualitative features of the equilibrium shape of a material void in a linearly elastic solid may be deduced from smoothness and convexity properties of the interfacial energy.

In addition, short time existence, uniqueness, and regularity for an anisotropic surface diffusion evolution equation with curvature regularization are proved in the context of epitaxially strained two-dimensional films. This is achieved by using the H^{-1}-gradient flow structure of the evolution law, via De Giorgiś minimizing movements. This seems to be the first short time existence result for a surface diffusion type geometric evolution equation in the presence of elasticity.

On a parabolic-hyperbolic system for contact inhibition of cell-growth

Danielle Hilhorst, University Paris-Sud, France

Abstract. We consider a parabolic-hyperbolic system of nonlinear partial differential equations which describes a simplified model for contact inhibition of growth of two cell populations. In this talk, we show that global solutions exist and that they satisfy a segregation property which reflects the inhibition mechanism: if the two populations are initially segregated – in mathematical terms this is translated into disjoint spatial supports of their densities – this property remains valid for all later times. To this purpose, we apply recent results on transport equations and Lagrangian flows.

Numerical experiments show that, for certain parameter values and for a large class of initial data, the large time behavior of solutions is described by a segregated traveling wave solution with positive wave speed c^*. We will show that for each c larger or equal than c^* there exists a traveling wave, but for each $c > c^*$, the cell densities are continuous and no longer segregated. We also show that the latter traveling waves cannot be the large time profile of the system of partial differential equations for a large class of initial functions. The structure of the traveling waves strongly resembles that of the scalar Fisher-KPP equation, where a special role is played by the traveling wave with minimal speed.

Finally we prove that as a parameter tends to infinity, one of the component becomes extinct while the other one converges to the solution of the Fisher-KPP equation.

This is joint work with Michiel Bertsch, Hirofumi Izuhara, Masayasu Mimura and Tohru Wakasa.

Front propagation in nonlinear diffusion equations and systems

Hiroshi Matano, University of Tokyo, Japan

Abstract. In this talk I will present some recent results on the spreading of fronts in nonlinear diffusion equations on $\mathbf{R}^N$. Here, by "spreading of fronts", I mean the phenomenon in which the level sets of a non-negative solution with compactly supported initial data expand outward and eventually cover the entire space as time tends to infinity.

In this talk, I will mainly focus on the following two different types of problems:

(1) Equations with multi-stable nonlinearity. (2) Reaction-diffusion systems of the prey-predator type. As for (1), it will be shown that the long time behavior of any solution can be described by what we call a "radially symmetric propagating terrace". This is joint work with Yihong Du.

As for (2), we will show that both the prey and the predator spread with certain definite speeds. Quite intriguingly, the spreading speed of the prey and that of the predator may be different in some cases. This is joint work with Arnaud Ducrot and Thomas Giletti.

I will also mention some other recent results on spreading fronts including my joint work with Fabio Punzo and Alberto Tesei on the front propagation in the hyperbolic space.

Solution of Leray's problem for stationary Navier-Stokes equations in plane and axially symmetric spatial domains

Konstantin Pileckas, University of Vilnius, Lithuania

Abstract. We study the nonhomogeneous boundary value problem for the Navier-Stokes equations of steady motion of a viscous incompressible fluid in arbitrary bounded multiply connected plane or axially-symmetric spatial domains. We prove that this problem has a solution under the sole necessary condition of zero total flux through the boundary. The problem was formulated by Jean Leray 80 years ago. The proof of the main result uses Bernoulli's law for a weak solution to the Euler equations.

This is the joint work with M. Korobkov and R. Russo

Weak solutions to Fokker-Planck equations and mean-field games

Alessio Porretta, Università di Roma Tor Vergata, Italy

Abstract. Mean-field games theory leads to the study of systems of PDEs coupling Fokker-Planck with viscous Hamilton-Jacobi equations. In case of local coupling, solutions are not known to be regular and a weak theory is needed. I will discuss the existence and uniqueness of weak solutions, standing on a new analysis of Fokker-Planck equations with L^2 drift. Applications include the optimal planning problem for the associated stochastic dynamics which, in PDE terms, consists in the exact controllability of Fokker-Planck equations.

Existence and regularity results for the heat flow of higher dimensional H-systems

Anna Verde, University Federico II of Naples, Italy

Abstract. In this talk we deal with the Cauchy-Dirichlet problem associated to the heat flow of the so-called H-system in dimension $n \geq 3$. The H-system has a natural geometric property: a solution to the H-system represents a hypersurface whose mean curvature is given by the real function H. After showing the existence of a "small" weak solution to the corresponding heat flow, we discuss the regularity of such a solution.

The results presented are obtained in collaboration with C. Leone and M. Misawa.

2. Short communications & Minisymposia

On the modulus of continuity of solutions to $p-$Laplacian type equations

Angela Alberico, IAC - CNR, Italy

Abstract. We deal with local solutions to a class of elliptic equations whose prototype is the $p-$Laplacian equation, with $2 \leq p \leq n$.
Although most of our results, which will appear in [ACS], are new for every $p \in [2, n]$, we mainly focus on the case when $p = n$, namely on the $n-$Laplacian equation

$$-\text{div}\,(|\nabla u|^{n-2}\nabla u) = f(x) \qquad \text{in } \Omega, \tag{1}$$

where Ω is an open set in $\mathbb{R}^n$, $n \geq 2$.
We exhibit the largest rearrangement invariant space to which f has to belong for every local solution u to (1) to be continuous in Ω.
Moreover, we find the optimal modulus of continuity of solutions to (1) when f belongs to a wide class of rearrangement invariant spaces, including Lorentz-Zygmund spaces $L^{r,q}(\log L)^{\alpha}(\Omega)$, and various standard Orlicz spaces. Specifically, we obtain estimates of the form

$$\|u\|_{\mathcal{C}^{0,\varphi}_{loc}(\Omega)} \leq C \left(\|f\|^{\frac{1}{n-1}}_{X(\Omega)} + \|\nabla u\|^{\frac{1}{n-1}}_{L^1(\Omega)} \right),$$

where $\mathcal{C}^{0,\varphi}_{loc}(\Omega)$ denotes the space of locally uniformly continuous functions with modulus of continuity φ, $X(\Omega)$ is a rearrangement invariant space, and C is a positive constant.
In particular, we complement and improve earlier results of [IO], [JKY] and [Te], and also of [FF] in the case when $p < n$.

(Joint work with A. Cianchi and C. Sbordone)

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Traveling waves
for nonlocal reaction diffusion equations (without comparison)

Matthieu Alfaro, University Montpellier 2, France

Abstract. We consider a nonlocal effect in the reaction term of classical reaction diffusion equations (Fisher-KPP case, ignition case, bistable case) so that the comparison principle does not hold. We review known facts and present recent developments on such issues.

The talk is based on joint works with J. Coville and G. Raoul.

On regularity properties of solutions to the hysteresis-type problem

Daria Apushkinskaya, Saarland University, Germany

Abstract. We consider equations with the simplest hysteresis operator at the right-hand side. Such equations describe the so-called processes "with memory" in which various substances interact according to the hysteresis law. In this talk some recent results concerning the optimal regularity of solutions are presented. Our arguments are based on quadratic growth estimates for solutions near the free boundary. The talk is based on several joint works with Nina Uraltseva.

On Asymptotic Behavior of Solutions to Higher Order Nonlinear Ordinary Differential Equations with Power Nonlinearity

Irina Astashova, Lomonosov Moscow State University, Russia

Abstract. For the equation

$$y^{(n)} = p\left(x, y, y', \dots, y^{(n-1)}\right) |y|^k \operatorname{sgn} y, \quad n \geq 2, \quad k > 0, \quad k \neq 1, \tag{2}$$

asymptotic behavior of its solutions is investigated.

New results about asymptotic behavior of solutions to equation (2) will be discussed, in particular, the existence of solutions with some special qualitative properties. Thus, for the equation

$$y^{(n)} + p_0 |y|^k \operatorname{sgn} y = 0, \quad n > 2, \quad k \in \mathbb{R}, \quad k > 1, \quad p_0 \neq 0, \tag{3}$$

the existence of oscillatory and non-oscillatory quasi-periodic in some sense solutions is proved.

Theorem 1. *For any integer $n > 2$ and real $k > 1$ there exists a non-constant oscillatory periodic function $h(s)$ such that for any $p_0 > 0$ and $x^* \in \mathbb{R}$ the function*

$$y(x) = p_0^{\frac{1}{k-1}} (x^* - x)^{-\alpha} h(\log(x^* - x)), \quad -\infty < x < x^*, \quad \alpha = \frac{n}{k-1}, \tag{4}$$

is a solution to equation (3).

The corollaries from this theorem for even and odd n and the results about the existence of solutions with given number of zeros in the given domain are also proved.

The same result is also obtained for equation (3) with $0 < k < 1$.

Theorem 2.([4]) *For $12 \leq n \leq 14$ there exists $k > 1$ such that equation* (3) *with $p_0 < 0$ has a solution $y(x)$ satisfying*

$$y^{(j)}(x) = (x^* - x)^{-\alpha - j} h_j(\log(x^* - x)), \quad \alpha = \frac{n}{k-1},$$
$$j = 0, 1, \dots, n-1,$$

where h_j are periodic positive non-constant functions on $\mathbf{R}$.

A similar result is also formulated for Kneser solutions, i. e. those satisfying $y(x) \to 0$ as $x \to \infty$ and $(-1)^j y^{(j)}(x) > 0$ for $0 \leq j < n$.

Earlier results to (2) can be found in [1], [2], [3].

These results and methods of proofs can be used in investigations of properties of nonlinear elliptic and parabolic partial differential equations.

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Lifespan estimates for solutions of pKirchhoff systems

Giuseppina Autuori, University Politecnica delle Marche, Italy

Abstract. In this talk we shall present some perturbed evolution systems governed by the p-Kirchhoff operator in bounded domains. These models are characterized by time dependent nonlinear driving forces and dynamic boundary conditions. The question of noncontinuation of maximal solutions will be discussed and some a priori estimates for the lifespan of solutions will be given.

On the regularity for variational inequalities and applications to equilibrium problems

Annamaria Barbagallo, University of Naples Federico II, Italy

Abstract. The aim of the talk is to study under which assumptions the continuity of solutions to infinite dimensional variational inequalities can be ensured and to apply these results to the most important equilibrium problems. The key tool in order to obtain the continuity is an appropriate use of the set convergence in Mosco-Kuratowskis sense and a generalized monotonicity assumption. As a matter of the fact, the most popular equilibrium problems under suitable assumptions admit a continuous solution. Furthermore in many problems the Lipschitz continuity can be ensured. Both continuity and Lipschitz continuity are very useful for computing equilibrium solutions.

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A quantitative modulus of continuity in the two-phase Stefan problem

Paolo Baroni, University of Uppsala, Sweden

Abstract. The (two-phase) Stefan problem is a well-known mathematical model for the evolution of the configuration of a substance which is changing phase; one should think, for instance, of a piece of very cold ice melting in water. In this talk I am going to show how, using classic tools as the weak Harnack inequality, one can provide an explicit, logarithmic-type modulus of continuity for local weak solutions; our method also allows to consider degenerate diffusions.

This is a joint work with T. Kuusi (Aalto University, Helsinki) and J. M. Urbano (University of Coimbra).

Perturbed p–Laplacian problems

Rossella Bartolo, University Politecnico di Bari, Italy

Abstract. The aim of this talk is to present some recent results about the existence and the multiplicity of solutions of the elliptic problem

$$(P) \qquad \begin{cases} -\Delta_p u \;=\; g(x,u) + \varepsilon h(x,u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $1 < p < +\infty$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, Ω is an open bounded domain of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $\varepsilon \in \mathbb{R}$ and h is just a continuous function, under different conditions on g.

(a) If $p = 2$ and g is asymptotically linear at infinity, suitable procedures and estimates allow us to prove that the number of distinct critical levels of the functional associated to the unperturbed problem is "stable" under small perturbations, in particular obtaining multiplicity results if g is odd, both in the non–resonant and in the resonant case. Moreover, if $p \neq 2$, g is asymptotically $(p-1)$–linear and $\varepsilon = 0$, we get the existence and the multiplicity of solutions of the quasilinear elliptic problem (P) by means of some abstract critical point theorems on Banach spaces and using two sequences of quasi–eigenvalues for the p–Laplacian operator.

(b) By using the so–called Bolle's method, we prove the existence of infinitely many solutions of (P) when $g(x,u) = |u|^{q-2}$, $p < q < \frac{pN(p+1)}{pN+N-p}$, $\varepsilon = 1$, $h(x,u) = h(x)$ and $u = \varphi$ on $\partial\Omega$, with $\varphi \in C^2(\overline{\Omega})$. Better results can be obtained in the radial case.

A model for the extreme wind speeds in hurricanes

Michiel Bertsch, University of Rome Tor Vergata, Italy

Abstract. We revisit a model for the laminarization and acceleration of shear flows due to the presence of suspended particles (in the case of hurricanes the suspended particles are water droplets). We analyze the equation for the flow velocity.

Joint work with Joost Hulshof (Amsterdam) and Valerij Prostokishin (Moscow).

Around Cournt-Nash equilibria via optimal transport

Adrien Blanchet, Université Toulouse 1, France

Abstract. We will review some recent results which we obtained in collaboration with F. Santambrogio and G. Carlier on Cournot-Nash equilibrium in a game with a continuum of players. We will prove that this equilibrium can be seen as the limit of Nash equilibria when the number of players goes to infinity and review some of the results which we obtained in the potential game case but also in a non-potential case.

Relaxation and Duality for L^∞ Mass Transportation Problems

Marian Bocea, Loyola University Chicago, USA

Abstract. The original mass transportation problem, formulated by Gaspard Monge in 1781, asks to find the optimal volume preserving map between two given sets of equal volume, where optimality is measured against a cost function given in integral form. After reviewing some classical aspects of the Monge problem, with an emphasis on Kantorovich's ideas based on relaxation and duality, I will describe an approach to develop a duality theory for the case of relaxed L^∞ cost functionals acting on probability measures with prescribed marginals.

Relations between the classical mountain pass theorem and local minima

Gabriele Bonanno, University of Messina, Italy

Abstract. The aim of this talk is to present a local minimum theorem for continuously Gâteaux differentiable functionals. Some remarks on the classical mountain pass theorem are also pointed out and its relations with local minima are highlighted. As a consequence, multiplicity results for nonlinear differential problems are obtained.

On some elliptic problems with variable exponents

Maria-Magdalena Boureanu, University of Craiova, Romania

Abstract. We are concerned with nonlinear elliptic problems involving generalized operators. The search for weak solutions to our problems is conducted in the framework of the variable exponent spaces. We rely on the critical point theory to establish existence, uniqueness and multiplicity results.

Symmetry breaking in a constrained Cheeger type isoperimetric inequality

Barbara Brandolini, Università degli Studi di Napoli "Federico II", Italy

Abstract. The study of the optimal constant $\mathcal{K}_q(\Omega)$ in the Sobolev inequality

$$\|u\|_{L^q(\Omega)} \leq \frac{1}{\mathcal{K}_q(\Omega)} \|Du\|(\mathbb{R}^n), \qquad 1 \leq q < 1^*,$$

for BV functions which are zero outside Ω and with zero mean value inside Ω, leads to the definition of a Cheeger type constant. We are interested in finding the best possible embedding constant in terms of the measure of Ω alone. We set up an optimal shape problem and we completely characterize, on varying the exponent q, the behaviour of optimal domains. Among other things we establish the existence of a threshold value $1 \leq \tilde{q} < 1^*$ above which the symmetry of optimal domains is broken. Several differences between the cases $n = 2$ and $n \geq 3$ are emphasized.

Nonlinear flow in porous media

Renata Bunoiu, University of Lorraine, Metz, France

Abstract. The flow on Newtonian fluids in porous media was extensively studied, in classical porous media as far as in porous media with a double periodic structure. No mathematical result is known about the flow of non-Newtonian fluids in domains with double periodicity. The aim of the talk is to give some results obtained in this last case and to discuss some open problems.

Optimal regularity for nonlinear elliptic and parabolic equations with nonstandard growth

Sun-Sig Byun, Seoul National University, Korea

Abstract. We discuss on an optimal regularity theory of the gradient of weak solutions for nonlinear elliptic and parabolic equations with nonstandard growth.

The weak inverse mapping theorem

Daniel Campbell, Charles University Prague, Czech Republic

Abstract. We prove that if a bilipschitz mapping f is in $W^{m,p}_{\mathrm{loc}}(\mathbb{R}^n, \mathbb{R}^n)$ then the inverse f^{-1} is also a $W^{m,p}_{\mathrm{loc}}$ class mapping. Further we prove that the class of bilipschitz mappings belonging to $W^{m,p}_{\mathrm{loc}}(\mathbb{R}^n, \mathbb{R}^n)$ is closed with respect to composition and multiplication without any restrictions on $m, p \geq 1$. These results can be easily extended to smooth n-dimensional Riemannian manifolds and further we prove a form of the implicit function theorem for Sobolev mappings.

Reinforcement for variational inequalities on fractal sets

Raffaela Capitanelli, University Roma Sapienza, Italy

Abstract. Reinforcement problems in the classical setting of regular domains were widely studied in conjunction with numerous applications. In this talk, I present some results on reinforcement for variational inequalities on fractal sets.

Uniform resolvent convergence for a strip with fast oscillating boundary

Giuseppe Cardone, Università del Sannio, Italy

Abstract. Given a planar infinite strip with a fast oscillating boundary we consider an elliptic operator assuming that both the period and the amplitude of the oscillations are small. On the oscillating boundary we impose Dirichlet, Neumann or Robin boundary condition. We describe the homogenized operator, establish the uniform resolvent convergence of the perturbed resolvent to the homogenized one, and prove the estimates for the rate of convergence. These results are obtained as the order of the amplitude of the oscillations is less, equal or greater than that of the period. It is shown that under the homogenization the type of the boundary condition can change.

Recent results in materials with memory

Sandra Carillo, Sapienza Università di Roma, Italy

Abstract. Challenging analytical problems are originated from applications; indeed, new materials are more and more widely studied since they are used in a variety of different environments. In particular, in recent years there has been a growing interest in smart material in general and, also, in materials with memory. Indeed, these materials exhibit the crucial physical property that their behaviour depends on time not only through the present time but also through their past history. This peculiarity leads, under the analytical point of view to study integro-differential model problems. This is the case of both rigid thermodynamics with memory as well as viscoelasticity. The model of a rigid linear heat conductor with memory is considered and, specifically, an evolution problem which describes the time evolution of the temperature distribution is studied. The attention is focussed on the thermodynamical state of such a rigid heat conductor which, according to the model proposed by Fabrizio, Gentili and Reynolds in 1998 and the constitutive equations therein, depends on the history of the material. When initial and boundary conditions are assigned, the model evolution problem is represented by an integro-differential one whose kernel depends on time through the present time as well as the past history of the material. A a consequence, the choice of suitable expressions of the minimum free energy and of the thermal work turns out to be crucial in this study. Indeed, the functional spaces where solutions are looked for, are obtained by the requirement to be meaningful both under the physical as well as the analytic viewpoint. Then, on application of existence and uniqueness results recently proved, conditions which guarantee solutions exponential decay at infinity are obtained. Furthermore, some existence, and, possibly uniqueness, results recently obtained in joint work with V. Valente and G. Vergara Caffarelli, in the case, in turn, of a magneto-viscoelasticity problem, or of a singular viscoelasticity problem are considered. Again, the choice of suitable free energies plays a key role.

Droplets spreading under contact line friction: scaling laws and existence result

Maria Chiricotto, TU Dresden, Germany

Abstract. This talk is concerned with the spreading of a liquid droplet on a plane solid surface in the regime of lubrication approximation. The focus is on effective conditions which relate the speed of the contact line (where liquid, solid and vapor meet) to the microscopic contact angle. One such condition has been recently proposed by Weiqing Ren and Weinan E [Phys. Fluids 19(2007)]: it includes into the model the effect of frictional forces which arise at the contact line from unbalanced components of the Young's stress, leading to an additional dissipation term in the energy balance. In lubrication approximation, the spreading of thin droplets may be modeled by a class of free boundary problems for fourth order nonlinear degenerate parabolic equations. We provide the analysis of the well-posedness, regularity, and qualitative properties of weak solutions. For speed-dependent contact angle conditions of rather general form, a matched asymptotic study is worked out, relating the macroscopic contact angle to the speed of the contact line. Here, well-posedness for a class of traveling-wave solutions is proved and used. The results have been obtained jointly with Lorenzo Giacomelli (La Sapienza - University of Rome).

Homogenization and correctors of a class of elliptic problems in perforated domains

Imen Chourabi, University of Rouen, France

Abstract. In this work, we study the homogenization and the correctors for a class of linear elliptic problems in a periodically perforated domain when the oscillating matrix field also depend on a weakly converging sequence. We prescribe a Dirichlet condition on the exterior boundary and a non homogeneous and nonlinear Robin condition on the boundary of the holes. Using the periodic unfolding method, we first derive the homogenized problem, then we study the convergence of the energy of the solution and the related corrector, which are the main results of this work. As a particular case, we obtain a corrector result for the laplacian with a linear nonhomogeneus Neumann condition on the hole, in the case of a non zero Neumann date with a zero average. This remained an open problem since the corresponding homogenized results given in [1].

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(Minisymposia: Homogenization and Multiple Scales, organized by Patrizia Donato).

Multiple positive solutions of nonlinear Schrödinger equations concentrating at a potential well

Silvia Cingolani, Politecnico di Bari, Italy

Abstract. I will present some recent results contained in a joint work with Louis Jeanjean and Kazunaga Tanaka.

We consider the singularly perturbed nonlinear Schrödinger problem

$$-\varepsilon^2\Delta u + V(x)u = f(u), \quad u > 0, \quad u \in H^1(\mathbb{R}^N) \tag{5}$$

where $V \in C(\mathbb{R}^N, \mathbb{R})$ and f is a nonlinear term which satisfies the so-called Berestycki-Lions conditions. We assume that there exists a bounded domain $\Omega \subset \mathbb{R}^N$ such that

$$m_0 \equiv \inf_{x\in\Omega} V(x) < \inf_{x\in\partial\Omega} V(x)$$

and we set $K = \{x \in \Omega \mid V(x) = m_0\}$. For $\varepsilon > 0$ small, we prove the existence of at least $\mathrm{cupl}(K) + 1$ solutions to (5) concentrating, as $\varepsilon \to 0$ around K, where $\mathrm{cupl}(K)$ denotes the cup-length of K defined with Alexander-Spanier cohomology.

Our approach is purely variational and it does not require uniqueness and non-degeneracy of the least energy solution of the limiting problem. Moreover f does not need to satisfy monotonicity assumptions and the search of solutions to (5) cannot be reduced to the study of the critical points of a functional restricted to a Nehari manifold.

A variational approach to the Brown-Ravenhall operator for the relativistic one-electron atom

Vittorio Coti Zelati, University Napoli Federico II, Italy

Abstract. We give an alternative characterization of the eigenvalues and eigenfunctions for the Brown-Ravenhall operator (operator related to the Dirac operator) in the case of a one-electron atom.

Kinetic formulation and uniqueness for scalar conservation laws with discontinuous flux

Graziano Crasta, Sapienza Università di Roma, Italy

Abstract. We prove a uniqueness results for BV solutions of scalar conservation laws with discontinuous flux in several space dimensions. Our method of proof is based on the notion of kinetic solution and on a careful analysis of the entropy dissipation along the discontinuities of the flux.

On the high regularity of solutions to the p-Laplacean boundary value problem in exterior domains

Francesca Crispo, Second University of Naples, Italy

Abstract. We consider the boundary value problem for the p-Laplacian system, $\in (1,2)$, in exterior domains. For suitable p and L^q-space, we furnish existence of high regular solutions u, that is solutions whose second derivatives belong to $L^q(\Omega)$. Hence, in particular, for $q > n$, we get $u \in C^{1,\alpha}(\overline{\Omega})$. The result is obtained under the same p-restriction given for the boundary value problem in bounded domains, and requiring that $q \geq p$.

Joint work with Carlo R. Grisanti (Università di Pisa) and Paolo Maremonti (Seconda Università degli Studi di Napoli)

Gradient regularity in Orlicz space for the gradient of solutions to quasilinear elliptic equations in the plane

Luigi D'Onofrio, Università degli Studi di Napoli "Parthenope"
(joint work with Linda Maria De Cave and Roberta Schiattarella)

Abstract. Recently many authors investigate the regularity of the gradient of solutions of the Dirichlet problem

$$\begin{cases} \operatorname{div} A(x, \nabla v) = f & \text{in } \Omega \\ v = 0 & \text{on } \partial\Omega \end{cases} \tag{6}$$

where Ω is a bounded domain in the plane, A satisfies the Leray-Lions type conditions and f belongs to a suitable Zygmund space (see [1] [2] [4]).

In this talk I will present a joint work with Linda Maria De Cave and Roberta Schiattarella in which we prove that, for any $\beta \geq 0$, if f belongs to $L(\log L)^{\frac{1}{2}}(\log\log\log L)^{\beta}$ there exists a unique solution $v \in W_0^{1,2}$ to (6) such that $|\nabla v| \in L^2(\log\log\log L)^{\beta}$.

The crucial and main tool in our arguments is the estimate

$$\|\nabla\varphi\|_{L^2(\log\log\log L)^{-\beta}} \leq C(\beta)\|\underline{\chi}\|_{L^2(\log\log\log L)^{-\beta}}$$

that we found for solutions of

$$\begin{cases} \operatorname{div} A(x, \nabla\varphi) = \operatorname{div} \underline{\chi} & \text{in } \Omega \\ \varphi \in W_0^{1,1}(\Omega) \end{cases}$$

by using a method due to [3].

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Transmission problems and time-changed diffusions on irregular domains

Mirko D'Ovidio, Sapienza University of Rome, Italy

Abstract. We consider time-changed diffusions on fractal domains driven by generators with discontinuous coefficients. The PDE's connections are investigated and in particular some results on elliptic problems with transmission conditions are presented.

Nonnegative solutions for a class of singular parabolic problems involving p-laplacian

Ida de Bonis, Sapienza Università di Roma, Italy

Abstract. We deal with the existence of nonnegative solutions to parabolic problems which are singular in the u variable whose model is

$$\begin{cases} u_t - \Delta_p u = f(x,t)(\frac{1}{u^\theta} + 1) & \text{in } \Omega \times (0,T) \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{in } \Omega. \end{cases}$$

Here Ω is a bounded open subset of $\mathbb{R}^N, N \geq 2$, $0 < T < +\infty$, $\theta > 0$, $\Delta_p u = -\mathrm{div}(|\nabla \mathrm{u}|^{\mathrm{p}-2}\nabla \mathrm{u})$ with $p > 1$.
As far as the data, we assume $f(x,t) \in L^r(0,T;L^m(\Omega))$, with $\frac{1}{r} + \frac{N}{pm} < 1$, $f(x,t) \geq 0$ a.e. in $\Omega \times (0,T)$ and $u_0(x) \geq 0$ a.e. in Ω.
We consider also the case where the right hand side depends on the gradient of the solution. In this last case the model of the right hand side is $F(x,t,u,\nabla u) = \frac{f(x,t)+D|\nabla u|^q}{u^\theta}$, with $\theta > 0$, $D > 0$, $1 < q < p$ and $f(x,t)$ as before.

Nonlinear elliptic equations with singular nonlinearities

Linda Maria De Cave, Sapienza Università di Roma, Italy

Abstract. In this talk we will present existence and regularity results for positive solutions of a nonlinear singular elliptic boundary value problem with homogeneous Dirichlet boundary conditions. The differential operator is like a p-laplacian ($1 < p < N$), and the equation is singular since the lower order term is $f(x)/u^\lambda$, where $f \geq 0$ belongs to some Lebesgue space and $\lambda > 0$. We will also give a nonexistence result if f is a nonnegative Radon measure concentrated on a set of (a suitable) zero capacity; for example, a Dirac mass. In the last part of the talk we will present existence results for a nonlinear variational elliptic system, in which one of the equations (or both) has a singularity as above.

Existence theory and targe time asymptotics for some nonlocal interaction equations and systems

Marco Di Francesco, University of Bath, United Kingdom

Abstract. I will report of a recent result on the existence, uniqueness, and large time behaviour of measures solutions of systems of nonlocal intearction equations with many species and mildly singular potentials in arbitrary space dimension (work in collaboration with S. Fagioli). The case of symmetric systems is treated via a generalization of the JKO/AGS theory for systems, whereas an implicit/explicit version of the JKO scheme allows for existence of weak measure solutions for non symmtric systems. I will also briefly report of a recent result in collaboration with M. Fornasier, D. Matthes, and J.-C. Huetter, on the large time emergence of nontrivial steady states for a one-dimensional equation arising from the so called 'kinetic dithering' problem.

Fractal pseudo-differential equation

Tomasz Dlotko, University of Silesia, Poland

Abstract. Using the technique of semilinear parabolic equations Cauchy's problem with fractional power $(-\Delta)^\alpha, \alpha \in (0,1)$:

$$\begin{cases} u_t + (-\Delta)^\alpha u = F(x,u,\nabla u),\ t>0,\ x \in \mathbb{R}^N, \\ u(0,x) = u_0(x),\ x \in \mathbb{R}^N, \end{cases} \tag{7}$$

was studied in [1, 2, 3]. The problem is popular nowadays in connection with the *anomalous diffusion* (e.g. [4]) extending the classical Laplacian diffusion.

Under various assumptions concerning the nonlinear term local and global solvability of (7) in both classical Sobolev spaces and in *locally uniform spaces* are discussed. We collect also several technical tools suitable in the studies of pseudo-differential equations. For particular F, using the *tail estimates technique*, a global attractor in $L^2(\mathbb{R}^N)$ was constructed in [2]. Paper [3] is devoted to sub-critical ($\alpha \in (\frac{1}{2}, 1)$) fractional Hamilton-Jacobi equation in locally uniform spaces.

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Optimality condition for weakly minimal solutions to infinite dimensional constrained vector equilibrium problems and application

Maria Bernadette Donato, University of Messina, Italy

Abstract. In this talk, we introduce a necessary optimality condition for weakly minimal solutions to the constrained vector equilibrium problems without requiring that the ordering cone which defines the inequality constraints has a nonempty interior. Furthermore, we provide an application which allows to formulate the infinite dimensional Lagrange multipliers rule for constrained vector optimization problems.

The two-scale approach to hydrodynamic limits for non-reversible dynamics

Manh Hong Duong, Eindhoven University of Technology, the Netherlands

Abstract. In a recent paper by Grunewald et.al., a new method to study hydrodynamic limits was developed for reversible dynamics. In this work, we generalize this method to a family of non-reversible dynamics. As an application, we obtain quantitative rates of convergence to the hydrodynamic limit for a weakly asymmetric version of the Ginzburg-Landau model endowed with Kawasaki dynamics. These results also imply local Gibbs behavior, following a method introduced in a recent paper by the second author.

This is a joint work with Max Fathi.

On some fractional abstract nonlinear differential equations with integrated semi-group

Mahmoud M. El-Borai and Khairia El-Said El-Nadi, Alexandria University, Egypt

Abstract. Let A be a linear closed operator defined on a dense set in a Banach space E to E. In this note it is supposed that A is the generator of α-times integrated semi group, where α is a positive number. The abstract Cauchy problem of the fractional nonlinear differential equation:

$$\frac{d^{\alpha}u(t)}{dt^{\beta}} = Au(t) + F(u,t),$$

with the initial condition $u_0 \in E$, is studied, where $0 < \beta \leq 1$, and F is a given abstract function. An application is given. Keywords and phrases: α-times integrated semi groups, abstract fractional nonlinear differential equations. 2000 Mathematics subject classifications: 47D60, 47D62, 35A05.

A stochastic model of the growth and diffusion of brain tumor cancer

Mahmoud M. El-Borai and Khairia El-Said El-Nadi, Alexandria University, Egypt

Abstract. The various treatment protocols such as surgery, radiation and chemotherapy can not effect a cure of patients with high grade brain tumors. We consider stochastic partial differential equations models, which uses patients data and brain scans to quantify the spatio-temporal growth of such brains tumors. The considered models are generalizations of some deterministic models. In the frame of thus stochastic models we are able to study the growth models for tumor cells under the influence of random perturbations. The Adomian method is used to study the nonlinear model.

Keywords and Phrases: Stochastic partial differential equations, brain cancer, Adomian. method
AMS Subject Classifications. 37C45, 92B05.

Global regularity and stability of solutions to elliptic equations with nonstandard growth

Michela Eleuteri, University of Firenze, Italy

Abstract. We study the regularity properties of solutions to elliptic equations similar to the p(x)-Laplacian. Our main results are a global reverse Hölder inequality, Hölder inequality up to the boundary and stability of solutions with respect to continuous perturbations in the variable growth exponent. We assume that the complement of the domain is uniformly fat in a capacitary sense. In the last part of the talk we discuss the same results for the corresponding obstacle problem.

Asymptotic behavior of the solutions of time dependent micromagnetism problem

Luisa Faella, Università degli Studi di Cassino e del Lazio Meridionale, Italy

Abstract. The aim is to describe the asymptotic behavior of the solutions of time dependent micromagnetism problem in a multi-domain consisting of two joined ferromagnetic thin films. Different regimes, depending on the limit of the ratio between the small thickness of the two films, will be distinguished.

Schauder regularity for linear and quasilinear equations structured on Hörmander vector fields

Maria Stella Fanciullo, University of Catania, Italy

Abstract. For a linear nonvariational equations structured on smooth Hörmander vector fields, with Hölder continuous coefficients, we prove a $C^{k.\alpha}$ regularity result. Moreover we deduce an analogous regularity result for the solutions of nonvariational quasilinear equations.

Optimization of the principal eigenvalue under mixed boundary conditions

Maria Antonietta Farina, University of Cagliari, Italy

Abstract. We investigate minimization and maximization of the principal eigenvalue of the Laplacian under mixed boundary conditions in case the weight has indefinite sign and varies in a class of rearrangements. Biologically, these optimization problems are motivated by the question of determining the most convenient spatial arrangement of favorable and unfavorable resources for a species to survive or to decline. We prove existence and uniqueness results, and present some features of optimizers.

Mappings of finite distortion and composition operators

Fernando Farroni, Università degli Studi di Napoli Federico II, Italy

Abstract. In this walk, we consider the action of the composition operator induced by a homeomorphism of finite distortion on several function spaces. Let Ω, Ω' be bounded domains of $\mathbb{R}^n$ and let $f : \Omega \to \Omega'$ be a homeomorphism in the Sobolev class $W^{1,1}_{\mathrm{loc}}(\Omega, \mathbb{R}^n)$. We will consider the case of a homeomorphism f having finite inner or outer distortion. We recall that f has *finite outer distortion* if its Jacobian $J_f = \det Df$ is strictly positive a.e. on the set where $|Df| \neq 0$. Similarly, we recall that f has *finite inner distortion* if its Jacobian $J_f = \det Df$ is strictly positive a.e. on the set where $|\mathrm{adj}\, Df| \neq 0$. Here, $\mathrm{adj}\, Df$ is the adjugate of the differential matrix Df of f. We will focus on the the regularity properties of the function $u \circ f^{-1}$ where $u : \Omega \to \mathbb{R}$ is a measurable function belonging to a given function space $X(\Omega)$. We will be interested in showing how the integrability and differentiability properties of $u \circ f^{-1}$ are affected by the choice of the space $X(\Omega)$ and by the assumptions on the homeomorphism f.

On the sharp effect of attaching a thin handle on the spectral rate of convergence

Veronica Felli, Università di Milano 'Bicocca', Italy

Abstract. Consider two domains connected by a thin tube: it is well known that the resolvent of the Dirichlet Laplacian is continuous with respect to the channel section parameter. This in particular implies the continuity of isolated simple eigenvalues and the corresponding eigenfunctions with respect to domain perturbation. Under an explicit nondegeneracy condition, we improve this information providing a sharp control of the rate of convergence of the eigenvalues and eigenfunctions in the perturbed domain to the relative eigenvalue and eigenfunction in the limit domain. This is a joint paper with L. Abatangelo and S. Terracini.

On the minimizers of trace inequalities in BV

Vincenzo Ferone, Università di Napoli Federico II, Italy

Abstract. It is well known that, for any given bounded domain $\Omega \subset \mathbb{R}^n$ with a "nice" boundary, $BV(\Omega)$ embeds in $L^1(\partial\Omega)$, in the sense that the total variation of a function u bounds the L^1 norm of $(u-c)$ through a constants K which depends on Ω. About c various choices can be made. We consider the cases where c is the median or the mean value of the trace of u over the boundary of Ω. We prove that balls achieve the least embedding constant K in both inequalities. Uniqueness of such minimizers is also discussed in details. Some of the tools used in the proof are: modified Cauchy area formula, characterization of sets of constant brightness, characterization of sets of constant projection.

Elliptic problems with hardy potential

Massimiliano Ferrara, Mediterranea of Reggio Calabria, Italy

Abstract. In this talk we present some existence results for certain singular elliptic Dirichlet problems involving the p-Laplacian. Precisely, starting from a weak lower semicontinuity result and by using the classical Hardy inequality, a critical point result for differentiable functionals is exploited, in order to prove the existence of a precise open interval of positive eigenvalues for which the treated problems admit at least one non-trivial weak solution.

This result is obtained in a recent paper in collaboration with G. Molica Bisci, Existence results for elliptic problems with Hardy potential, Bull. Sci. Math.(in press).

On the eigenvalues of a boundary value problem with a parameter

Alexey Filinovskiy, N.E. Bauman Moscow State Technical University, Russia

Abstract. We consider the eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in} \quad \Omega, \quad \frac{\partial u}{\partial \nu} + \alpha u = 0 \quad \text{on} \quad \Gamma, \tag{8}$$

where $\Omega \subset R^n$, $n \geq 2$, is a bounded domain with boundary $\Gamma \in C^2$. By ν we denote the outward unit normal vector to Γ, α is a real parameter. There is a sequence of eigenvalues $\lambda_1(\alpha) < \lambda_2(\alpha) \leq \dots$ of the problem (8) enumerated according to their multiplicities with $\lim\limits_{k\to\infty} \lambda_k(\alpha) = +\infty$. Also, we consider the sequence of eigenvalues $0 < \lambda_1^D < \lambda_2^D \leq \dots$ of the Dirichlet eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on} \quad \Gamma.$$

with $\lim\limits_{k\to\infty} \lambda_k^D = +\infty$. Note that the eigenvalues $\lambda_1(\alpha)$ and λ_1^D are simple and the corresponding eigenfunctions $u_{1,\alpha}(x)$ and $u_1^D(x)$ are positive. We study the behavior of $\lambda_k(\alpha)$ for large values of α.

Theorem 1. *The eigenvalues have the following properties:*

i) $\lambda_k(\alpha)$, $k = 1, 2, \dots$, *are continuous functions of* α *and*

$$\lambda_k(\alpha_1) \leq \lambda_k(\alpha_2) \leq \lambda_k^D, \quad \alpha_1 < \alpha_2;$$

ii) $\lambda_1(\alpha)$ *is a concave function of* α*:*

$$\lambda_1(\beta\alpha_1 + (1-\beta)\alpha_2) \geq \beta\lambda_1(\alpha_1) + (1-\beta)\lambda_1(\alpha_2), \quad 0 < \beta < 1;$$

iii) $\lambda_1(\alpha)$ *is differentiable and*

$$\lambda_1'(\alpha) = \frac{\int_\Gamma u_{1,\alpha}^2\, ds}{\int_\Omega u_{1,\alpha}^2\, dx} > 0.$$

Theorem 2. *The eigenvalues* $\lambda_k(\alpha)$, $k = 1, 2, \dots$, *satisfy the estimates*

$$0 \leq \lambda_k^D - \lambda_k(\alpha) \leq C\alpha^{-1} \left(\lambda_k^D\right)^2, \quad \alpha > 0,$$

where the constant C *does not depends on* k*.*

Theorem 3. *The following estimate*

$$\liminf_{\alpha\to-\infty} \frac{\lambda_1'(\alpha)}{-\alpha} \geq 1$$

holds.

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Global Existence of Renormalized Solutions to Entropy-Dissipating Reaction-Diffusion Systems

Julian Fischer, University of Zurich, Switzerland

Abstract. Consider a single reversible chemical reaction with mass-action kinetics. The corresponding reaction-diffusion equation then formally satisfies an entropy inequality. Nevertheless, in general global existence of solutions for the reaction-diffusion equation has remained an open problem due to the possibly steep growth of the reaction terms. We propose a notion of renormalized solutions for such equations and succeed in proving global existence of solutions for general initial data and coefficients. Our notion of renormalized solutions reduces to the usual notion of weak solutions if the reaction terms are integrable.

A dynamical approach to Laplace and p-Laplace equations

Matteo Franca, Università Politecnica delle Marche, Ancona, Italy

Abstract. We discuss the existence and the asymptotic behavior of positive radial solutions of the following equation, in the whole $\mathbb{R}^n$:

$$\Delta u(\mathbf{x}) + f(u, |\mathbf{x}|) = 0,$$

and of its generalization to the p-Laplace case. We are mainly interested in structure results in the case where f exhibits both subcritical and supercritical behavior. We want to emphasize the big difference existing between the case where f is subcritical for u large and supercritical for u small, from the case where f is supercritical for u large and subcritical for u small. In fact this latter case is subject two several bifurcation phenomena which do not appear in the former case. Finally we explain a surprising contribution on the critical case, i.e. $f(u, r) = k(r)u^{\frac{n+2}{n-2}}$: we show that the existence of positive critical points of k is sufficient to guarantee the existence of ground states with fast decay, but if we have a maximum we have uniqueness while if we have a minimum we may have an arbitrarily large number of such solutions. The results are obtained using Fowler transformation which enables us to exploit dynamical system techniques.

Ferroelectric thin structures

Antonio Gaudiello, Università degli Studi di Cassino e del Lazio Meridionale, Italy

Abstract. I present some results obtained in collaboration with Kamel Hamdache (Ecole Polytechnique, Palaiseau, France). Starting from a classical non-convex and nonlocal 3D-variational model of the electric polarization in a ferroelectric material, via an asymptotic process we obtain 1D and 2D-variational models. Depending on the initial boundary conditions, the limit problem can be either nonlocal or local.

Pointwise bounds and arbitrarily large solutions for semilinear elliptic systems

Marius Ghergu, School of Mathematical Sciences University College Dublin, Ireland

Abstract. We study the behavior around the origin of C^2 positive solutions $u(x)$ and $v(x)$ to the system

$$\begin{aligned} 0 \le -\Delta u \le f(v) \\ 0 \le -\Delta v \le g(u) \end{aligned} \quad \text{in } B_1(0) \setminus \{0\} \subset \mathbb{R}^n, n \ge 2,$$

where $f, g : (0, \infty) \to (0, \infty)$ are continuous functions. We provide optimal conditions on f and g such that the above system admits pointwise bounds around the origin. In dimension $n = 2$ we show that this property holds if $\log^+ f$ or $\log^+ g$ grow at most linearly at infinity. In dimension $n \ge 3$ and under the assumption $f(t) = O(t^p)$, $g(t) = O(t^q)$ as $t \to \infty$, $(p, q \ge 0)$, we obtain a new critical curve that optimally describes the existence of such pointwise bounds.

This talk is based on a joint work with S.D. Taliaferro (Texas A&M) and I.E. Verbitsky (Missouri).

Existence and homogenization for a singular problem through rough surfaces

Daniel Giachetti, Universit'a di Roma la Sapienza, Italy

Abstract. The paper deals with existence and homogenization for elliptic problems with lower order terms singular in the u-variable (u is the solution) in a cylinder Q in $\mathbb{R}^N$. A rapidly oscillating interface exists in Q. The interface has a periodic microstructure and it is situated in a small neighbourhood of a hyperplane which separates two connected composite components of Q. At the interface we suppose the following transmission conditions: (i) the flux is continuous, (ii) the jump of a solution at the interface is proportional to the flux through the interface.

Existence of Lagrange multipliers for the elastic-plastic variational inequality

Sofia Giuffrè, Mediterranea University of Reggio Calabria, Italy

Abstract. Aim of this talk is to present recent results related to the elastic-plastic torsion problem in different cases, with a particular attention to its relationship with the obstacle problem and to the existence of Lagrange multipliers. These results are obtained by means of the strong duality theory. Finally we search, in the case of a ball, solutions to the elastic-plastic torsion problem and associated Lagrange multipliers of radial type.

Drift-diffusion models for heterostructures in photovoltaics

Annegret Glitzky, Weierstrass Institute, Germany

Abstract. The talk discusses an electronic model for solar cells taking into account heterostructures with active interfaces and energy resolved volume and interface trap densities. The model consists of continuity equations for electrons and holes with thermionic emission transfer conditions at the interface and of ODEs for the trap densities with energy level and spatial position as parameters, where the right hand sides contain generation-recombination as well as ionization reactions. This system is coupled with a Poisson equation for the electrostatic potential.

We investigate the thermodynamic correctness of the model and motivate a priori estimates for the solutions to the evolution system. Moreover, existence and uniqueness results of weak solutions of the problem are presented. For the existence proof regularized problems have to be solved and bounds of the corresponding solution not depending on the regularization level have to be verified. Finally we comment on the corresponding stationary problem.

Asymptotic expansions for anisotropic singular perturbations problems

Senoussi Guesmia, Qassim University, Saudi Arabia

Abstract. We describe the asymptotic expansions for the solutions to the elliptic, anisotropic singular perturbations problems far from the boundary layer as well as on the whole domain. The coefficients of the development are defined as solutions of an iterative sequence of elliptic problems. Since the perturbations are only taken in some directions we have to study the smoothness of the coefficients, in order to complete the definition of this sequence of problems.

This is a joint work with S. Azouz (U. O.Ebouaghi, Algeria)

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Existence result for a nonlinear problem involving critical exponents and with weight

Rejeb Hadiji, University Paris Est Creteil, France

Abstract. We prove that a non vanishing boundary data gives existence result for a non linear pde problem with weight.

Some convergence results of parabolic problems on large non-cylindrical domains

Soumia Harkat, Larbi Ben M'hidi, Oum El Bouaghi, Algeria

Abstract. We are interested to study the asymptotic behaviour of an evolution problem defined on a non-cylindrical domain. The state variable domain is becoming unbounded if the time goes to infinity. Sufficient conditions are formulated in order to insure the convergence of the solution of some parabolic problems. The rate of convergence is also taken into account with respect to the growth rate of the domains.

This is a joint work with (S. Guesmia)

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Long-time behaviour of nonlocal Partial Differential Equations

Marta Herrera Cobos, Universidad de Sevilla, Spain

Abstract. The modeling and analysis of chemical, biological and physical processes by partial differential equations via evolution parabolic equations has been well known in the scientific literature for a long time, and even today it is a research topic very much considered for their applications.
On the other hand, much attention has been paid to nonlocal problems over the last few decades. The main reason is that many real situations are better modeled even though it is much more difficult to deal with such nonlocal operators.
In this contribution we will discuss on a parabolic equation with nonlocal diffusion. The existence and uniqueness of solution will be stated. Our main aim will be to study the asymptotic behaviour of the solutions. To do this we will work within the framework of dynamical systems.

This talk is based on a joint work with Tomás Caraballo and Pedro Marín-Rubio.

Stationary States and Asymptotic Behaviour of Aggregation Models with Nonlinear Local Repulsion

Yanghong Huang, Imperial College London, United Kingdom

Abstract. We consider a continuum aggregation model with nonlinear local repulsion given by a degenerate power-law diffusion with general exponent. The steady states and their properties in one dimension are studied both analytically and numerically, suggesting that the quadratic diffusion is a critical case. The focus is on finite-size, monotone and compactly supported equilibria. We also investigate numerically the long time asymptotics of the model by simulations of the evolution equation. Issues such as metastability and local/ global stability are studied in connection to the gradient flow formulation of the model.

Eigenvalues of the Laplacian in a domain with a thin tubular hole

Shuichi Jimbo, Hokkaido University, Japan

Abstract. I consider a singular domain variation and the characterization of the eigenvalues of the Laplacian under several boundary condition. I deal with a singularly perturbed domain, which is given by removing a thin tubular neighborhood of a lower dimensional manifold and the eigenvalue problem of the Laplacian. The case of the Dirichlet B.C has been studied more than 20 years ago, but other cases have not been studied well yet. I talk about a perturbation formula of the eigenvalue for Neumann and Robin B.C.

Existence of positive solutions of a semilinear elliptic equation with a dynamical boundary condition

Tatsuki Kawakami, Osaka Prefecture University, Japan

Abstract. We consider the initial value problem for a semilinear elliptic equation with a dynamical boundary condition in the half space, which the space dimension is greater or equal to 2. In this talk we prove that there is a critical exponent for the existence of positive solutions. Furthermore, in the supercritical case we show that small solutions behave asymptotically like suitable multiples of the Poisson kernel. Moreover we determine the optimal slow decay at spatial infinity for initial data giving rise to global bounded positive solutions.

This is based on some joint works with Marek Fila (Comenius University) and Kazuhiro Ishige (Tohoku University).

Strong solution to irreversible diffusion equation and application to crack propagation model

Masato Kimura, Kanazawa University, Japan

Abstract. We consider a nonlinear diffusion equation with irreversible property and construct a unique strong solution by using implicit time discretization. A new regularity estimate for the obstacle problem is established and is used in the construction of the strong solution. An application to a crack propagation model is also presented.

Partial regularity for minimizers of singular energy functionals

Olivier Kneuss, University of Zurich, Switzerland

Abstract. We study the partial regularity of minimizers for certain singular functionals in the calculus of variations, with application to liquid crystal models .

This is a joint work with Craig Evans and Hung Tran.

L^p Liouville theorems for Kolmogorov-Fokker-Planck equations

Alessia Elisabetta Kogoj, University of Bologna, Italy

Abstract. We show several Liouville-type theorems for subharmonic functions on Lie groups in R^{N+1}.

Our results apply in particular to the heat operator on Carnot groups and to Kolmogorov-Fokker-Planck operators in R^{N+1}.

Boundary design for fast diffusion across fractal boundaries

M. Rosaria Lancia, University of Sapienza, Italy

Abstract. It is by now well known that non stationary diffusion across the boundary of irregular domains is enhanced with respect to the diffusion across regular boundaries. Up to now the prototypes of irregular boundaries are selfsimilar fractals such as the Koch curve. We will consider the case in which the prefractal boundaries change according to the iteration. The resulting boundary is a non self similar set, the so called asymmetric (pre)fractal mixture. This is very important from the point of view of applications because it allows to "design" the boundary in order to enhance the diffusion or the absorption phenomena with respect to the location of the heat source.

We will present some a priori error estimates for the numerical approximation as well as some numerical test cases.

This results are obtained in collaboration with Massimo Cefalo.

This research was partially supported under the Grant NSF DMS-1109356, and it was carried on while the first autor was visiting Worcester Polytechnic Institute.

Homogenization of diffusion problems with a nonlinear interfacial resistance

Kim Hang Le Nguyen, University of Rouen, France

Abstract. In this work, we consider a stationary heat problem on a two-component domain with an ε-periodic imperfect interface, on which the heat flux is proportional via a nonlinear function to the jump of the solution, and depends on a real parameter (gamma).

Homogenization and corrector results for the corresponding linear case have been proved in [P. Donato, K. H. Le Nguyen, R. Tardieu, The Periodic Unfolding Method for a Class of Imperfect Transmission Problems, J. Math. Sciences, 176 (6) (2011), 891-927], by adapting the periodic unfolding method to the case of a two-component domain. Here, we first prove, under natural growth assumptions on the nonlinearities, the existence and the uniqueness of a solution of the problem. Then, we study, using the periodic unfolding method, its asymptotic behavior. In order to describe the homogenized problem, we complete some convergence results concerning the unfolding operators obtained in the linear case and investigate the limit behaviour of the unfolded Nemytskii operators associated to the nonlinear terms.

According to the values of gamma, we have different limit problems. The most relevant case is that gamma = 1, where the homogenized matrix differs from that of the linear case, and is described in a more complicated way, via a nonlinear function involving the correctors.

Gradient bounds and large solutions for some nonlinear elliptic equations

Tommaso Leonori, University of Carlos III de Madrid, Spain

Abstract. In this talk we consider the problem of constructing the so-called "large solutions" (i.e. solutions that blow up at the boundary) for a nonlinear elliptic equation that involves the p-laplacian with the presence of a first order lower order term. Gradient bounds for this class of equations are also discussed.

Influence of boundary conditions on the qualitative property of a reaction diffusion equation

Bendong Lou, Tongji University, China

Abstract. We study a reaction diffusion equation $u_t = u_{xx} + f(u)$ $(x \in [0, h(t)])$ with Robin boundary condition $u(0,t) = bu_x(0,t)$ and with a Stefan free boundary condition at $x = h(t)$. When f is an unbalanced bistable nonlinearity we prove a trichotomy result on the long time behavior of the solutions, that is, any solution converges either to 0 (i.e. vanishing), or an active solution (i.e. spreading), or a ground state $V(x - y(t))$ with finite or infinite shift $y(t)$ (i.e. transition). In the last case, we show that $y(t) \to z$ for some real z when b is large, and $y(t) = A \ln t + B + o(1)$ for some A, B depending on b and f when b is small.

Non-smooth critical point theory on closed convex sets and applications

Salvatore A. Marano, University of Catania, Italy

Abstract. The existence of multiple critical points for a locally Lipschitz functional F on a closed convex subset C of a Banach space X is investigated. The problem of finding extra conditions under which critical points for F on C turn out to be critical on X is also addressed. Some applications concerning elliptic variational-hemivariational inequalities are also presented.

Polyconvexity and existence theorems in nonlinear shell theory

Cristinel Mardare, University of Pierre et Marie Curie, France

Abstract. The notion of polyconvexity has been introduced by John Ball in 1977 to establish the existence of minimizers for a class of non-convex functionals appearing in nonlinear elasticity theory. We discuss a similar notion of polyconvexity, this time for functionals appearing in nonlinear shell theory, which eventually allows to establish the existence of minimizers for for a class of such functionals.

Nonlinear Saint-Venant compatibility conditions and the intrinsic approach for nonlinearly elastic plates

Sorin Mardare, University of Rouen, France

Abstract. This work has been done in collaboration with Philippe G. Ciarlet of City University of Hong Kong. We study the Kirchhoff-Love model of nonlinearly elastic plates and we adopt an "intrinsic" approach by considering as the main unknowns (instead of the deformation itself) two 2x2 symmetric matrix fields derived from the deformation of a plate subjected to some volume forces. This approach allows us to describe the set of admissible forces for the pure traction problem, i.e. to find the necessary conditions that the forces must satisfy in order to have existence of a minimizer for the associated energy functional. The key role is played here by the rigidity properties of the new intrinsic unknowns. We also establish necessary and sufficient nonlinear compatibility conditions of Saint-Venant type for these unknowns. More precisely, these conditions ensure that two symmetric matrix fields satisfying them derive from a deformation.

Analysis on the Sierpinski Gasket

Giovanni Molica Bisci, University of Reggio Calabria, Italy

Abstract. In this talk, we will report some recent results contained in the paper: G. Molica Bisci and V. Radulescu, A characterization for elliptic problems on fractal sets, Proc. Amer. Math. Soc. (in press).

On the boundedness of some classes of elliptic operators with discontinuous coefficients

Sara Monsurrò, University of Salerno, Italy

Abstract. A potential estimate approach in the achievement of some a priori estimates for the solutions of certain classes of non-divergence form elliptic equations having discontinuous coefficients is proposed.

Existence and stability of standing wavefronts in FitzHugh-Nagumo equations

Yoshihisa Morita, Ryukoku University, Japan

Abstract. We will deal with the FitzHugh-Nagumo equations, a well-known reaction-diffusion model of activator-inhibitor type. In similar to Allen-Cahn equation, a balanced condition for the potential induced from the reaction term is assumed for studying planar standing wavefronts. We first establish the existence of the standing wavefronts by a variational argument if the diffusion rates of activator and inhibitor are in an appropriate range. Then we show the stability in the one-dimensional space with an additional condition. The stability in a cylinder domain is also ensured. This is a joint work with Chao-Nien Chen and Shih-Yin Kung.

Liouville type theorems for anisotropic degenerate elliptic problems

Luisa Moschini, University of Rome La Sapienza, Italy

Abstract. We establish L^∞ Liouville type theorems for anisotropic degenerate elliptic equations in divergence form on strips $S = R^{n-1} * (-1, 1)$ where $x = (x', \lambda)$; that is for equations whose model is

$$\mathrm{div}_{x'}(w_1 \nabla_{x'} \sigma) + \partial_\lambda (w_1 w_2 \partial_\lambda \sigma) = 0,$$

where the weight functions $w_i(x', \lambda)$ are locally positive and bounded. We prove our Liouville theorems by means of an oscillation decrease argument which generalizes De Giorgi's technique under appropriate conditions on the weight functions w_i; the key ingredient being the existence of a positive unbounded supersolution (or "almost-supersolution") of the equation close to the degeneration set ∂S. A special interest is devoted to the case $w_1 = (1 - |\lambda|)$ and $w_2 = (1 - |\lambda|)^2$, for which the Liouville type theorem we prove entails an alternative proof of the positive answer to a famous conjecture of De Giorgi in any space dimension under the additional assumption that the zero level set of the solution is a Lipschitz graph. Such a positive answer has been already given by Barlow, Bass and Gui in 2000 through a probabilistic approach and by Caffarelli and Cordoba in 2006, by a similar oscillation decrease argument which makes anyhow strongly use of the semilinear Ginzburg Landau equation. The above particular choice of weight functions also gives a further motivation for our approach since we prove that no global Harnack elliptic inequality holds true on the sets on which we prove an oscillation decrease result.

The results presented in this talk have been recently obtained in collaboration with X. Cabré.

Laplace and Stokes problems in a punctured domain

Wei Müller, University of Zurich, Switzerland

Abstract. We study the Laplace equation in a two dimensional periodical punctured domain. We show the existence of a solution and study the convergence of the solution when the disc shrinks. We study also the analogue for the Stokes problem. There we need some approximation by divergence free functions which vanish on the discs. The technique we used is based on results of Bogovskii.

Joint work with M. Chipot, G. Planas and J. Robinson.

Methods for pattern formation in models for phase separation

Andreas Muench, University of Oxford, United Kingdom

Abstract. We discuss aspects of pattern formation arising from the Cahn-Hilliard equation for phase separation in confined geometries and for models involving surface diffusion.

A mathematical formulation of the discrete variational derivative method for dissipative systems

Ken-Ichi Nakamura, Kanazawa University, Japan

Abstract. We study a numerical method for nonlinear partial differential equations with energy dissipation property. The discrete variational derivative method has been used to obtain some special numerical schemes that have the same dissipation property in a discrete sense. We give a new mathematical formulation of the discrete variational derivative method and propose a simple procedure for the derivation of dissipative numerical schemes.

On two fractional Laplacians

Alexander Nazarov, St. Petersburg Steklov Mathematical Institute and St. Petersburg State University, Russia

Abstract. Let Ω be a bounded domain with Lipschitz boundary. We compare two natural types of fractional Laplacians $(-\Delta)^s$, namely, the "Navier" and the "Dirichlet" ones, for $0 < s < 1$.

Lemma 1. *The domains of forms $((-\Delta_\Omega)^s_N u, u)$ and $((-\Delta_\Omega)^s_D u, u)$ coincide with $\widetilde{H}^s(\Omega) = \{u \in W^s_2(\mathbb{R}^n) : \operatorname{supp} u \subset \overline{\Omega}\}$.*

Theorem 1. *The difference $(-\Delta_\Omega)^s_N - (-\Delta_\Omega)^s_D$ is positive definite and positivity preserving. Namely, for $u \in \widetilde{H}^s(\Omega)$, we have*

$$((-\Delta_\Omega)^s_N u, u) \geq ((-\Delta_\Omega)^s_D u, u). \tag{9}$$

If, in addition, $u \geq 0$, then we have in the sense of distributions

$$(-\Delta_\Omega)^s_N u \geq (-\Delta_\Omega)^s_D u. \tag{10}$$

If $u \not\equiv 0$ then relations (9) and (10) hold with strict sign.

Theorem 2. *If $u \in \widetilde{H}^s(\Omega)$, then*

$$((-\Delta_\Omega)^s_D u, u) = \inf_{\Omega' \supset \Omega} ((-\Delta_{\Omega'})^s_N u, u)$$

(the infimum is taken over the set of smooth bounded domains in $\mathbb{R}^n$).

Theorem 3. *Assume $n > 2s$ and set $2^*_s = \frac{2n}{n-2s}$. (This is a restriction only for $n = 1$.) Then*

$$\inf_{\substack{u \in \widetilde{H}^s(\Omega) \\ u \neq 0}} \frac{((-\Delta_\Omega)^s_N u, u)}{\|u\|^2_{L_{2^*_s}(\Omega)}} = \inf_{\substack{u \in \widetilde{H}^s(\Omega) \\ u \neq 0}} \frac{((-\Delta_\Omega)^s_D u, u)}{\|u\|^2_{L_{2^*_s}(\Omega)}}.$$

This talk is based on a joint paper with Roberta Musina, see [1].

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On the ω-limit set of a nonlocal evolution problem

Thanh Nam Nguyen, Université Paris-Sud, France

Abstract. We consider an initial value problem for a nonlocal differential equation with a bistable nonlinearity and discuss about its ω-limit set. We show that for a large class of initial functions, the ω-limit set contains exactly one element, which has the form of a step function, and takes at most two values.

This is joint work with Danielle Hilhorst, Hiroshi Matano and Hendrik Weber.

Dilute emulsions with surface tension

Grigor Nika, Worcester Polytechnic Institute, United Kingdom

Abstract. We consider an emulsion formed by two Newtonian fluids, one fluid being dispersed in the other under the form of identical droplets in the presence of surface tension. The flow is at low Reynolds numbers. Depending on the relation between the characteristic size of the droplets and the distance between them, we identify at first order two types of effective behavior: a Brinkman type law or a Stokes type law. The results are obtained using Γ-convergence methods.

Quasilinear elliptic equations and systems with Morrey data

Dian K. Palagachev, Politecnico di Bari, Italy

Abstract. We will present some recent results regarding regularity of weak solutions to quasilinear elliptic equations and systems with Morrey-type lower order terms supporting controlled non-linear growths. Under very general assumptions on the non-smooth boundary of the underlying domain, global essential boundedness and continuity of the weak solutions will be discussed.

Homogenization of Systems of Partial Differential Equations

Antonio Jesús Pallares Martín, Universidad de Sevilla, Spain

Abstract. In the present work we study the homogenization of systems of partial differential equations on a bounded regular domain $\Omega \subset \mathbb{R}^N$. Thanks to the maximum principle, in the case of equations it suffices to assume weak compactness of the coefficients in L^1 to prove that the limit problem is of the same type. In our case, because of the lack of a "maximum principle-like" result in systems, we need to use tools of Γ-convergence and compensated compactness in order to obtain suitable conditions which allow us to establish the homogenization result.

This is a joint work with Marc Briane, Juan Casado-Díaz and Manuel Luna-Laynez.

Spectral problems in porous media with Robin type boundary conditions on the cavities and larger adsorption parameters

M. Eugenia Pérez, University of Cantabria, Spain

Abstract. We address asymptotics for spectral problems posed in periodically perforated domains along a plane. The operator under consideration is the Laplacian, and the spectral problem is posed in a three dimensional domain Ω, outside the cavities. The boundary conditions are of the Dirichlet type on the boundary of Ω and of the Robin type on the boundary of the cavities. The periodicity of the structure is ε; it is a small parameter that converge towards zero. The size of the cavities can be of the same order of magnitude of ε, namely $O(\varepsilon)$, or much smaller than ε, namely $o(\varepsilon)$. Also a large ε-dependent parameter (adsorption constant) arises in the Robin conditions. Depending on the different values/relations between the three parameters (periodicity, size of cavities and adsorption constant) different homogenized problems are obtained: both critical sizes for cavities and critical relations for parameters are provided (cf. [1-3]). Extensions to other operators and geometrical configurations are outlined (cf. [4-5]).

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A Variational Method for a Boundary Value Nonlinear Problem

Carmen Perugia, Università del Sannio, Italy

Abstract. In this talk we will discuss a variational method for the study of the asymptotic behavior of a vectorial nonlinear problem in a domain of $\mathbb{R}^n$ periodically perforated by two families of holes. Every element of one family moves towards an element of the other family (it could also lie on this one). We will assign different conditions on the boundary of the holes of each family and study the asymptotic behavior in dependence also of the approaching speed.

Maximal solutions for singular elliptic and parabolic problems

Francesco Petitta, Sapienza, Università di Roma, Italy

Abstract. In this talk I will discuss some recent results concerning the existence and uniqueness of large/maximal solutions for singular elliptic and parabolic problems. In the elliptic case a general condition on the absorption term of the $1-$Laplace elliptic equation is given in order to get both existence and uniqueness. This condition can be considered as the correspondent Keller-Osserman condition for the $p-$Laplacian in the case $p = 1$. In the framework of parabolic p-Laplace equations we will show how existence and uniqueness of suitable large solutions can be obtained for singular problems (i.e. $1 \leq p < 2$) without the presence of lower order absorption terms.

On the uniqueness of the positive Cauchy problem for a class of ultraparabolic operators

Sergio Polidoro, University Modena and Reggio Emilia, Italy

Abstract. We prove the uniqueness of the positive Cauchy problem for a family of degenerate hypoelliptic operators of the form of "sum of squares of vector fields plus a drift term". Our proof relies on the construction of suitable Harnack chains, on the characterization of the extremal rays of the cone of non-negative solutions and on the application of Choquet theorem.

This is a joint work in collaboration with Alessia Kogoj and Yehuda Pinchover.

Qualitative behavior of solutions of doubly nonlinear parabolic equations with a linear reaction term

Maria Assunta Pozio, Sapienza Università di Roma, Italy

Abstract. We study the qualitative properties of explicit self–similar solutions for the doubly nonlinear diffusion equation with a positive linear reaction term. For this problem $u \equiv 0$ is an unstable stationary solution and we prove that its unstable manifold has infinite Hausdorff dimension, even if the spatial domain is bounded, provided that the degree of homogeneity of the diffusion operator is larger then one.

Indeed explicit self–similar solutions exist that tend to 0 as $t \to -\infty$ while their support shrinks to an arbitrarily chosen point or a ball, a line, an hyperplane, an infinite cylinder and so on.

These results are in marked contrast with the case of non-degenerate semilinear equations and had already been proved by H. Matano and the author, for $p = 2$.

Power law approximation under differential constraints

Francesca Prinari, University of Ferrara, Italy

Abstract. We study the Γ-convergence of the power-law functionals

$$F_p(V) = \Big(\int_\Omega f^p(x, V(x)) dx \Big)^{1/p},$$

as p tends to $+\infty$ and $V \in L^\infty(\Omega; \mathbb{M}^{d\times N}) \cap \operatorname{Ker} \mathcal{A}$ where $\mathcal{A}$ is a constant-rank partial differential operator. We show that the Γ-limit is represented in the supremal form

$$F(V) = \operatorname*{ess\,sup}_{x\in\Omega} \tilde{f}(x, V(x)) \tag{11}$$

and we give an explicit representation formula for the function $\tilde{f}$. Moreover we discuss the weak* lower semicontinuity of functionals of the form (11) when $V \in L^\infty(\Omega; \mathbb{M}^{d\times N}) \cap \operatorname{Ker} \mathcal{A}$.

Allen-Cahn Approximation of Mean Curvature Flow on Riemannian Manifolds

Fabio Punzo, Università degli Studi di Milano, Italy

Abstract. The talk is concerned with the parabolic Allen-Cahn equation on complete, not necessarily compact Riemannian manifolds, with Ricci curvature bounded from below. For quite general initial conditions, we show nonpositivity of the limit discrepancy, and we obtain an almost local monotonicity formula. Then such results will be used to prove that the energy measure converges to a limit Radon measure, which evolves according to the Mean Curvature Flow, in the Brakke's sense.

From a general viewpoint, we shall discuss similarities and important differences with respect to the same question in the Euclidean space.

All results have been obtained in collaboration with A. Pisante (Sapienza Università di Roma).

A qualitative property for a class on elliptic problems on the Sierpinski gasket

Vicentiu Radulescu, Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract. In this talk, we will report some recent results contained in the paper *G. Molica Bisci, V. Radulescu, A characterization for elliptic problems on fractal sets, Proc. Amer. Math. Soc., in press.*

We are concerned with a characterization theorem on the existence of a non-zero strong solution for elliptic equations on the Sierpinski gasket. More generally, the validity of our result can be checked studying elliptic equations defined on self-similar fractal domains whose spectral dimension is less than 2. Our theorem can be viewed as an elliptic version on fractal domains of a recent contribution obtained by Ricceri for a two-point boundary value problem.

On the properties of the solutions of integro-differential equations arising in the theory of heat conduction

Nadezhda Rautian, Lomonosov Moscow State University, Russia

Abstract. The main goal of our work is studying of the asymptotic behavior of the solutions of Gurtin-Pipkin type integro-differential equations on the base of spectral analysis of their symbols. For this purpose, strong solutions of these equations are represented as a sum of terms corresponding to the real and nonreal parts of the spectrum of the operator functions that are the symbols of these equations. The resulting representations are new for the given class of integro-differential equations. Since the Gurtin-Pipkin type integro-differential equations arise in numerous applications, it is reasonable and natural to consider such equations with operator coefficients in a Hilbert space (abstract integro-differential equations), which, if necessary, can be specified as integro-differential equations with partial derivatives with respect to spatial variables.

(Joint report with Victor Vlasov)

Variational convergence of fractal energy forms in varying Hilbert spaces

Valerio Regis Durante, University Roma 3, Italy

Abstract. We consider the energy functional associated to a Venttsel problem in a 3D fractal domain and its approximating energy functionals in the corresponding prefractal domains. To study the asymptotic behaviour of the energies, the natural functional setting is that of varying Hilbert spaces. We prove the convergence of the approximating energies to the limit (fractal) one in the Mosco-Kuwae-Shioya sense. This in turn will imply the convergence of the semigroups in a suitable sense. These results are obtained in collaboration with M.R. Lancia and P. Vernole.

Existence of solutions for non level-convex problems in the supremal form

Ana Margarida Ribeiro, Universidade Nova de Lisboa, Portugal

Abstract. It is well known that lower semicontinuity of functionals in the supremal form is related to the level-convexity of the supremand. We address the problem of existence of solutions for Dirichlet boundary problems in the lack of this convexity condition relating it with some differential inclusion problem. This is a joint work with E. Zappale.

Sudden directional diffusion: counting and watching facets

Piotr Rybka, The University of Warsaw, Poland

Abstract. We study the parabolic problem of the form

$$u_t = (\mathcal{L}(u_x))_x \quad \text{in } (0,b)\times(0,T).$$

The characteristic trait of function $\mathcal{L}$, which we consider is that it is increasing and it has jump(s). A jump of $\mathcal{L}$ at p_0 leads to creation of facets with slope p_0.

We concentrate on two instances of $\mathcal{L}$:
1) $\mathcal{L}(p) = \operatorname{sgn}(p+1) + \operatorname{sgn}(p-1)$;
2) $\mathcal{L}(p) = \operatorname{sgn} p + \epsilon p$.

The first case appears to admit infinite oscillations. We address this issue, showing that we have always only a finite number of facets with non-zero curvature. Moreover, with the help of the comparison principle for viscosity solutions we estimate the extinction time.

The second case is interesting because it is an instant of two types of competing diffusion. Counting facets becomes a more subtle job. We also show that their number is decreasing.

Asymptotic behaviour of the solutions of some nonlocal problems involving p-Laplace equations depending on the L^p-norm of the gradient

Tetiana Savitska, University of Zurich, Switzerland

Abstract. We are studying a class of nonlinear nonlocal diffusion problems associated with a p-Laplace-type operator, where a nonlocal quantity is present in the diffusion coefficient. More precisely, we consider the problem of finding $u = u(x,t)$ weak solution to

$$\begin{cases} u_t - \operatorname{div} a(\|\nabla u\|_p^p)|\nabla u|^{p-2}\nabla u = f & \text{in } \Omega\times(0,T), \\ u = 0 & \text{on } \Gamma\times(0,T), \\ u(\cdot,0) = u_0 & \text{in } \Omega, \end{cases} \tag{12}$$

where Ω is a bounded open set of $\mathbb{R}^n$, $n \geq 1$ with Lipschitz boundary Γ. We assume

$$a \text{ is continuous, } a(\xi) > 0,\ \forall \xi \in \mathbb{R}. \tag{13}$$

By $|\cdot|_p$ we denote the $L^p(\Omega)$-norm, $1 < p < +\infty$ and we assume

$$f = f(x) \in W^{-1,q}(\Omega) := \left(W_0^{1,p}(\Omega)\right)^*,\ u_0 \in W_0^{1,p}(\Omega)\cap L^2(\Omega),\ \frac{1}{p}+\frac{1}{q} = 1. \tag{14}$$

We address the issues of existence and uniqueness for the parabolic setting. Then we study the asymptotic behaviour of the solution for large time. For this purpose we introduce and investigate in details the associated stationary problem. Moreover, we show that the solutions of the stationary problem are also critical points of the energy functional

$$E(u) = \frac{1}{p}A\left(\int_\Omega |\nabla u|^p dx\right) - \langle f, u\rangle \tag{15}$$

with

$$A(z) = \int_0^z a(s)ds \tag{16}$$

and $\langle\cdot,\cdot\rangle$ denoting the pairing between $W^{-1,q}(\Omega)$ and $W_0^{1,p}(\Omega)$. We make a classification of its critical points and find a global minimizer for this energy functional. Furthemore, we consider the issue of stability of these critical points.

This is a joint work with M. Chipot (University of Zurich, Switzerland).

Bi-Sobolev map with zero Jacobian almost everywhere

Roberta Schiattarella, University of Naples Federico II, Italy

Abstract. For $n \geq 3$, we construct a bi–Sobolev map f such that $J_f = 0$ a.e. and $J_{f^{-1}} = 0$ a.e.. It follows that f maps a set of full measure to a null set and a remaining null set to a set of full measure. We also show that such a pathological homeomorphism cannot exist in dimension $n = 2$.

This is a joint work with Luigi D'Onofrio and Stanislav Hencl.

A fractional Cahn-Hilliard equation

Giulio Schimperna, University of Pavia, Italy

Abstract. In this talk I will present some recent results on a Cahn-Hilliard equation characterized by diffusion operators of fractional order. The equation is settled in a smooth bounded domain of $\mathbb{R}^3$ and complemented with homogeneous Dirichlet boundary conditions of "solid" type. In particular, I will discuss existence, uniqueness and regularity of weak solutions, singular limits, existence of stationary states, and long-time behavior of solution trajectories. These results have been obtained in collaboration with Goro Akagi (Kobe) and Antonio Segatti (Pavia).

Directional splitting for nonlinear fourth-order PDEs in imaging applications

Carola-Bibiane Schönlieb, University of Cambridge, United Kingdom

Abstract. We present directional operator splitting schemes for the numerical solution of a fourth-order, nonlinear partial differential evolution equation which arises in image processing. This equation constitutes the H^{-1}-gradient flow of the total variation and represents a prototype of higher-order equations of similar type which are popular in imaging for denoising, deblurring and inpainting problems. The efficient numerical solution of this equation is very challenging due to the stiffness of most numerical schemes. We show that the combination of directional splitting schemes with implicit time-stepping provides a stable and computationally cheap numerical realisation of the equation.

This is joint work with Bertram Düring and Luca Calatroni.

Ground-state solutions for a pseudo-relativistic Hartree equation with an external potential

Simone Secchi, Università di Milano-Bicocca, Italy

Abstract. We prove the existence of a positive solution to the equation $\sqrt{-\Delta + m^2}u + Vu = \left(W * |u|^\theta\right)|u|^{\theta-2}u$ in the space $\mathbb{R}^N$, with $m > 0$, V an external potential function and W a radially symmetric convolution kernel. Finally, we discuss the asymptotic behavior of the solution at infinity

Functions of least gradient

Sergio Segura de Leon, University of Valencia, Spain

Abstract. Let $\Omega \subset \mathbf{R}^N$ be a bounded open set whose boundary is Lipschitz–continuous. Given a function $h \in L^1(\partial\Omega)$, we define the functional

$$\Phi_h(u) = \int_\Omega |Du| + \int_{\partial\Omega} |u-h|\, d\mathcal{H}^{N-1}\,, \qquad u \in BV(\Omega)\,.$$

We consider the minimization of Φ_h and its (formal) Euler– Lagrange equation:

$$\begin{cases} -\mathrm{div}\left(\dfrac{Du}{|Du|}\right) = 0\,, & \text{in } \Omega\,; \\ u = h\,, & \text{on } \partial\Omega\,. \end{cases} \tag{17}$$

We prove that, for every $h \in L^1(\partial\Omega)$, there exists $u \in BV(\Omega)$ which is solution of both, the minimization problem and problem (17). We also give an example showing that the solution, in general, is not unique when $h \notin C(\partial\Omega)$.

On Some nonlocal fractional equations

Raffaella Servadei, Università della Calabria, Italy

Abstract. Fractional and non-local operators appear in concrete applications in many fields such as, among the others, optimization, finance, phase transitions, stratified materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic flows, multiple scattering, minimal surfaces, materials science and water waves. This is one of the reason why, recently, non-local fractional problems are widely studied in the literature.

Aim of this talk will be to present some recent results for nonlocal problems driven by the fractional Laplace operator $(-\Delta)^s$, which (up to normalization factors) may be defined as

$$-(-\Delta)^s u(x) = \int_{\mathbb{R}^n} \frac{u(x+y)+u(x-y)-2u(x)}{|y|^{n+2s}} dy, \quad x \in \mathbb{R}^n.$$

These results were obtained through variational and topological methods and extend the validity of some theorems known in the classical case of the Laplacian to the non-local fractional framework.

Global gradient estimates in weighted spaces for parabolic operators

Lubomira G. Softova, Second University of Naples, Italy

Abstract. We deal with the regularity problem for linear, second order parabolic equations and systems in divergence form with measurable data over non-smooth domains. Problems like this arise in the modeling of composite materials and in the mechanics of membranes and films of simple non-homogeneous materials which form a linear laminated medium. Assuming partial BMO smallness of the coefficients and Reifenberg flatness of the boundary of the underlying domain, we develop a Caldeón–Zygmund type theory for such parabolic operators in the settings of the weighted Lebesgue spaces. As consequence of the main result, we get regularity in parabolic Morrey scales for the spatial gradient of the weak solutions to the considered problem.

Time-delayed instabilities in complex Burgers equations

Marta Strani, University of Milano Bicocca, Italy

Abstract. For Burgers equations with real data and complex forcing terms, N. Lerner, Y. Morimoto and C.J. Xu proved that only analytical data generate local C^2 solutions. These instabilities are however not observed numerically; rather, numerical simulations show an exponential growth only after a delay in time. We argue that numerical diffusion is responsible for this time delay, and we show that the introduction of a small $\mathcal{O}(\varepsilon)$ viscous term in the equation can imply uniform bounds in time $\mathcal{O}(\sqrt{\varepsilon})$. To this aim we consider the viscous complex Burgers equations in the torus with small viscosity. We show that initial data $u_0(x) = a\,(x/\varepsilon)$ with large frequencies $\mathcal{O}(1/\varepsilon)$ generate solutions that are bounded in time $\mathcal{O}(1)$, before exhibiting an exponential growth in time.

These results have been obtained in collaboration with Benjamin Texier.

Measure-Valued Solutions of Forward-Backward Parabolic Equations

Alberto Tesei, Sapienza Università di Roma, Italy

Abstract. We study different pseudoparabolic regularizations of forward-backward parabolic equations of Perona-Malik type (joint work with M. Bertsch and F. Smarrazzo).

On the Allen-Cahn-Keller-Segel equations

Takeo Ushijima, Tokyo University of Science, Japan

Abstract. The Dictyostelium Discoideum, a kind of slime mold, is known to have its characteristic life cycle. In the early stage of this life cycle, they produce spiral patterns and stream patterns. To understand the mechanism of the pattern change from the spiral to stream, we introduce two model equations which we call FitzHugh-Nagumo-Keller-Segel equations and Allen-Cahn-Keller-Segel equations. In this talk, we will explain the relation between these two models and show some mathematical results for the Allen-Cahn-Keller-Segel equations. We will also show the results of numerical experiment for these two models.

This is a joint work with Danielle Hilhorst and Masayasu Mimura.

Effects of herd behavior on population interactions

Ezio Venturino, University of Torino, Italy

Abstract. The possible dynamics of an ecosystem composed of wild herbivores that gather together in herds and are subject to the attacks of predators are presented. The basic model differs from the classical Lotka-Volterra system because it explicitly incorporates the effects of predation on the prey that occupy the outermost positions in the herd. As a consequence, new phenomena arise.

Suspensions with Random Velocity-Dependent Interfacial Forces

Bogdan Vernescu, Worcester Polytechnic Institute, United Kingdom

Abstract. We study particulate flows or suspensions of solid particles in a viscous incompressible fluid in the presence of highly oscillatory, velocity dependent, surface forces. The flow at a small Reynolds number is modeled by the Stokes equations coupled with the motion of rigid particles. The objective is to perform homogenization for the given suspension and obtain an equivalent description of a homogeneous (effective) medium and determine the effective viscosity and the macroscopic effect of the surface forces.

A Mixed type system in the Koch snowflake domain

Maria Agostina Vivaldi, "Sapienza" Università di Roma, Italy

Abstract. In this talk we present existence, uniqueness and regularity results in weighted Sobolev spaces for solutions of mixed type systems on polygonal non-convex domains. Moreover we consider a sequence of mixed type systems on pre-fractal domains approximating the Koch snowflake domain and we discuss uniform estimates and asymptotic behavior of the solutions.

On the properties of the solutions of integro-differential equations arising in the theory of heat conduction

Victor Vlasov, Lomonosov Moscow State University, Russia

Abstract. The main goal of our work is studying of the asymptotic behavior of the solutions of Gurtin-Pipkin type integro-differential equations on the base of spectral analysis of their symbols. For this purpose, strong solutions of these equations are represented as a sum of terms corresponding to the real and nonreal parts of the spectrum of the operator functions that are the symbols of these equations. The resulting representations are new for the given class of integro-differential equations. Since the Gurtin-Pipkin type integro-differential equations arise in numerous applications, it is reasonable and natural to consider such equations with operator coefficients in a Hilbert space (abstract integro-differential equations), which, if necessary, can be specified as integro-differential equations with partial derivatives with respect to spatial variables.

(Joint report with Nadezhda Rautian)

Pulses and waves for nonlocal reaction-diffusion equations

Vitaly Volpert, University Lyon 1, France

Abstract. We will discuss recent results on the existence, stability and dynamics of pulses and waves for nonlocal reaction-diffusion equations. In the context of population dynamics they describe nonlocal and global consumption of resources and can be used to model emergence and evolution of biological species.

The wavelength of the contact line instability in dewetting rims

Barbara Wagner, Technische Universität Berlin, Germany

Abstract. We consider models that describe the dewetting of thin films, and in particular address the contact line instability which arises for unsteady non-uniform base states.

Well-posedness of degenerate Cauchy problems

Maria Wehowski, TU Dresden, Germany

Abstract. In this talk we consider the Cauchy problem

$$(M^{\frac{1}{2}}\partial M^{\frac{1}{2}} + B)(u) \ni M^{\frac{1}{2}} f,$$
$$(M^{\frac{1}{2}} u)(0^+) = x_0.$$

Here, ∂ denotes the time derivative, B is a multivalued maximal monotone operator in a Hilbert space H, M is a selfadjoint, continuous, monotone and possibly degenerate operator in H and $f \in L^1(0,T;H)$. Provided that $M + B$ is strongly monotone we develop a solution theory. In order to achieve existence and uniqueness results we introduce a time discretization of the problem and show that the limit of the approximate solutions is the solution of the Cauchy problem.

Relaxation results for integral functionals depending on two vector fields and applications to imaging and optimal design

Elvira Zappale, University of Salerno, Italy

Abstract. I will present some recent results dealing with relaxation and integral representation of integral functionals depending on a couple of vector fields. These models find applications in the context of imaging problems and optimal design.

Some obstacle problems for non-coercive operators

Gabriella Zecca, University of Naples "Federico II", Italy

Abstract. We study obstacle problems related to non-linear elliptic operator of the type

$$\mathrm{div}[\mathcal{A}(x,\nabla u)+\mathcal{B}(x,u)]$$

where $\mathcal{A}:\Omega\times\mathbb{R}^N\to\mathbb{R}^N$ satisfies standard *growth conditions* and $\mathcal{B}:\Omega\times\mathbb{R}\to\mathbb{R}^N$verifies, for all $t\in\mathbb{R}$, $|B(x,t)|\leqslant b(x)|t|$ with b in the Marcinkiewicz space $L^{N,\infty}(\Omega)$. We prove existence, uniqueness and regularity results for such problems without assuming the smallness of the $L^{N,\infty}$-norm of the function b. Therefore, we are dealing with problems that are in general not coercive.

Those results are obtained in collaboration with L. Greco and G. Moscariello.

Entropic flows, stochastic perturbations and microscopic models

Johannes Zimmer, University of Bath, United Kingdom

Abstract. As starting point, different ways to link some PDEs of diffusive type to particle models will be reviewed. Typically, the PDE will be related to a minimization problem of a large deviation rate function from probability.

The underlying microscopic process contains more information, notably fluctuations around the minimum state described by the deterministic PDE. Can stochastic terms be derived which model this additional information, in a way that is compatible to the limit passage via large deviations (and the geometric structure, such as the Wasserstein setting)? This question will be investigated for linear diffusion, and existence of a corresponding (nonlinear stochastic) equation will be analyzed.

This is joint work with Rob Jack and Shangjiang Guo.