Book of Abstracts

International Symposium on Applied Analysis
in Honour of the 65th Birthday of Michel Chipot and his retirement
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Degenerate Parabolic Equations

Herbert Amann, University of Zurich, Switzerland

Abstract. It is shown that degenerate parabolic equations are uniformly regular if they are studied on suitable Riemannian manifolds. This leads to optimal well-posedness results for degenerate problems in weighted Sobolev spaces.

Complex Ginzburg–Landau equation with absorption: existence, uniqueness and localization properties

Stanislav Antontsev, CMAF, University of Lisbon, Portugal

Abstract. Let $\Omega \subset \mathbf{R}^n$ be a bounded domain with Lipschitz-continuous boundary Γ and $Q_T = \Omega \times (0, T]$. We consider the following initial boundary value problem for complex function $u = \mathrm{Re}\ u + i\,\mathrm{Im}\,u$

$$e^{-i\gamma} u_t = \triangle u - a(x,t)\,|u|^{\sigma-2}\,u + f(x,t),\ 1 < \sigma < 2, -\pi/2 < \gamma < \pi/2, \tag{1}$$

$$u(x,0) = u_0(x),\ x \in \Omega;\ u|_{\Gamma_T} = 0,\ \Gamma_T = \partial\Omega \times (0,T). \tag{2}$$

Here a, f and u_0 are given complex value functions such that

$$0 \le |a| < \infty, f \in L^2(Q_T), u_0 \in L^2(\Omega).$$

Usually (1) is called as the time dependent complex Ginzburg-Landau (CGL). CGL with $\gamma = 0$ reduces to the well-known nonlinear heat equation $u_t - \Delta u = -|u|^{\sigma-2}u$. For $\gamma = \pm\pi/2$ CGL becomes the well-known nonlinear Schröodinger equation. CGL is very important as a model for the study of the pattern formation and the onset of instabilities in non equilibrium fluid dynamic systems. The mathematical study of the time dependent CGL equation can be found in many papers (see for example [2]). In this talk we prove existence of a global weak solution for (1),(2) applying the Galerkin approximation method. Unfortunately, we cannot prove a uniqueness result for the weak solutions. We prove also some localization properties for the solutions and finite time extinction in special cases. Part of the results can be found in [1].

Joint work with J. P. Dias and M. Figueira.

References

[1] S. N. Antontsev, J. P. Dias and M. Figueira, *Complex Ginzburg-Landau equation with absorption: existence, uniqueness and localization properties*, J. Math. Fluid Mechanics, (2013).

[2] T. Cazenave, F. Dickstein, and F. B. Weissler, *Finite-Time Blowup for a Complex Ginzburg–Landau Equation*, SIAM J. Math. Anal., 45 (2013), pp. 244–266.

Existence results for quasilinear elliptic eigenvalue problems in unbounded domains

Giuseppina Autuori, Università Politecnica delle Marche, Italy

Abstract. In this talk we shall present existence results for a nonlinear eigenvalue problem depending on a real parameter, driven by a weighted p-Laplacian operator and involving subcritical nonlinearities in unbounded domains, under Robin boundary conditions. Different approaches and methods will be presented, according to the size of the parameter, as well as to the assumptions on the nonlinearities, which could be either of Ambrosetti–Rabinowitz type or of Szulkin–Weth type.

These results are contained in a joint work with P. Pucci and C. Varga.

On the problem of prescribing the Jacobian determinant

Bernard Dacorogna, EPFL, Switzerland

Abstract. I will discuss the problem of existence, uniqueness and regularity for the equation $\det Du = f$.

Rate of Convergence to Separable Solutions of the Fast Diffusion Equation

Marek Fila, Comenius University, Slovakia

Abstract. We study the asymptotic behaviour near extinction of positive solutions of the Cauchy problem for the fast diffusion equation with a subcritical exponent. We show that separable solutions are stable in some suitable sense by finding a class of functions which belong to their domain of attraction. For solutions in this class we establish optimal rates of convergence to separable solutions.

This is a joint work with Michael Winkler.

Optimal estimates on free boundary propagation for the thin-film equation

Julian Fischer, University of Zurich, Switzerland

Abstract. We present a method for the derivation of lower bounds on free boundary propagation for the thin-film equation, one of the most prominent examples of a higher-order degenerate parabolic equation. In particular, we obtain sufficient conditions for instantaneous forward motion of the free boundary, upper bounds on waiting times, as well as lower bounds on asymptotic propagation rates. Our estimates coincide (up to a constant factor) with the previously known reverse bounds and are therefore optimal. To the best of our knowledge, these results constitute the first lower bounds on free boundary propagation for any higher-order degenerate parabolic equation. Our technique is based on new monotonicity formulas for solutions to the thin-film equation, which are combined with a differential inequality argument due to Chipot and Sideris. It turns out that our method is not restricted to the thin-film equation, but also applicable to other higher-order parabolic equations like quantum drift-diffusion equations.

On a waiting time phenomenon for the stochastic porous media equation

Günther Grün, University of Erlangen-Nürnberg, Germany

Abstract. We formulate a criterion on initial data which guarantees that solutions to stochastic porous media equations with linear multiplicative noise exhibit a waiting time phenomenon almost surely. Up to a logarithmic factor, it coincides with the optimal criterion known from the deterministic setting. A novel iteration technique and stochastic counterparts of weighted integral estimates used in the deterministic setting – these are the key ingredients of our approach, which may be modified to prove basic results on finite speed of propagation, too.

This is joint work with Julian Fischer (Zürich).

Asymptotic analysis of anisotropic singular perturbations

Senoussi Guesmia, Qassim University, Saudi Arabia

Abstract. We analyze here the asymptotic behaviour of the solutions to partial differential equations when some coefficients of the principal part (not all) of the differential operator become very small. This is what we call an anisotropic singular perturbations problem. As a typical example, we consider on $\Omega = (0,1) \times (0,1)$ a diffusion problem where the diffusion velocity is very small in the x_1 direction

$$\begin{cases} -\varepsilon^2 \partial^2_{x_1} u_\varepsilon - \partial^2_{x_2} u_\varepsilon = f & \text{in } \Omega, \\ u_\varepsilon = 0 & \text{on } \partial\Omega. \end{cases}$$

We are then interested in the limit behaviour of u_ε when $\varepsilon \to 0$ and the natural candidate is u_0 solution to

$$\begin{cases} -\partial^2_{x_2} u_0 dx = f & \text{in } (0,1)\,, \\ u_\varepsilon = 0 & \text{on } \{0,1\}\,. \end{cases}$$

Note that here the variable x_1 plays a role of a parameter. In this context and for some close problems we deal with different issues and questions related to the above convergence.

On Shape Optimization for Free Boundary Problems

Helmut Harbrecht, University of Basel, Switzerland

Abstract. In this talk, the solution of a Bernoulli type free boundary problem by means of shape optimization is considered. Four different formulations are compared from an analytical and numerical point of view. By analyzing the shape Hessian in case of matching data it is distinguished between well-posed and ill-posed formulations. A nonlinear Ritz–Galerkin method is applied for the discretization of the shape optimization problem. In case of well-posedness, existence and convergence of the approximate shapes is proven. In combination with a fast boundary element method efficient first and second order shape optimization algorithms are obtained.

Nehari manifolds for positive and compact support solutions to some semilinear elliptic problems related with nonlinear Schrödinger equations

Jesus Hernandez, University Autonoma de Madrid, Spain

Abstract. We study existence of positive and compact support solutions to a class of semilinear elliptic equations related with nonlinear Schrdinger equations.

First we prove existence of non-negative solutions by using a Nehari manifold argument. We also prove that solutions are actually positive for a bounded interval of values of a parameter and then study the existence of compact support solutions with the help of a Pohozaev identity. Bifurcation at infinity and the asymptotic behavior of the solution branches are studied as well.

This is joint work with J. I. Díaz and Y. Ilyasov.

Regularity analysis for deterministic second-order linear Kolmogorov partial differential equations

Arnulf Jentzen, ETH Zürich, Switzerland

Abstract. In this talk we analyze whether deterministic second-order linear Kolmogorov partial differential equations (PDEs) with smooth coefficients preserve regularity in the sense that solutions are smooth if the initial value function is smooth. In the first part of this talk we give an example of a second-order linear Kolmogorov PDE with a globally bounded and smooth drift coefficient, a constant diffusion coefficient and a smooth initial function with compact support such that the unique globally bounded viscosity solution of the PDE is not even locally Hölder continuous and, thereby, we disprove the existence of globally bounded classical solutions of this PDE. In the second part of this talk we present sufficient conditions to ensure that the Kolmogorov PDEs preserve regularity.

Eigenvalues of the Laplace–Beltrami operator on a spherical cap and related topics

Yoshitsugu Kabeya, Osaka Prefecture University, Japan

Abstract. We consider the distribution of the eigenvalues of the Laplace–Beltrami operator on a spherical cap when the cap covered almost the whole unit sphere. Using the information of the eigenvalues, we discuss a nonlinear elliptic problem on a spherical cap and the structure of the solution set.

On the potential theory of the farthest distance function

Bernd Kawohl, Universität zu Köln, Germany

Abstract. I report on joint work with C. Nitsch and G. Sweers. We were able to give a partial proof of a conjecture of Laugesen and Pritsker, according to which a measure of a set E related to the farthest point distance function to E is always bounded by the measure for a ball, as soon as E has more than one point. In two dimensions the bound is sharp for sets of constant width, but in higher dimensions this is no longer the case.

Quasi-Variational Inequality Arising from Models of Economic Growth

Nobuyuki Kenmochi, Bukkyo University, Japan

Abstract. In this talk we propose a regional economic growth model, with technological development in the prescribed knowledge-field. It is pointed out by many economists that a technological inovation often brings a big change in the production system, namely it enables to have a big output by a small labor force. This is a very important?point in respect that the labor force will not be expected enough in our future (by aging society or decline in population). Each production system selects some technologies obtained in the common knowledge-field and makes use of them to progress in the production. Our model consists of a parabolic partial differential inclusion describing the economic growth in a region and a parabolic variational (or quasi-variational) inclusion of the unknown-dependent subdifferentials describing the evolution of knowledge-technology orderparameter. In this talk some results on existence and large-time behaviour are given.

On a cross-diffusion PDE system

Robert Kersner, University of Pecs, Hungary

Abstract. The natural generalization of parabolic systems includes the use of cross-diffusion, for which the flow ("flux") is written as

$$J_k = -\sum D_{kj}(u)\partial_x u_j.$$

Cross-diffusion models have been considered in a number of studies in physical (plasma physics), chemical (dynamics of electrolyte solutions) , and biological (cross-diffusion transport) systems. The same refers to population dynamics and ecological (forest age-structure dynamics) studies. The Burridge–Knopoff model was used to describe interactions between tectonic plates in seismology. Mathematical models with cross-diffusion have been extensively employed in the past decade to gain insight into the mechanisms of tumor growth and development. I will be interested in a twoequation system with general "flux" and the reaction terms are of competition type. I shall speak mainly on existence and stability of nonconstant stationary solutions and mention some recent numerical results too.

Comments about mass transport and systems

David Kinderlehrer, Carnegie Mellon University, United States

Abstract. The first attempt to use mass transport methods to formulate questions about systems of evolution equations was, to my knowledge, the work of Chipot, Kinderlehrer, and Kowalczyk (2003). We bring the story up to date with a discussion of the Poisson-Nernst-Planck system, joint work with Leonard Monsaingeon and Xiang Xu.

Strong and metric regularity of generalized equations in constrained optimization

Diethard Klatte, University of Zurich, Switzerland

Abstract. Regularity concepts for equations and inclusions appearing in constrained optimization play an important role both for the sensitivity analysis of solutions and the convergence theory of solution methods. In this talk, we first discuss the classical notions of strong regularity and metric regularity of generalized equations (inclusions) and then present known and recent characterizations of these regularities with respect to constraint sets and critical points of constrained nonlinear optimization problems. In particular, we are interested in conditions under which metric and strong regularity for critical points coincide. Further we show that the latter property implies the non-degeneracy of constraints and hence the uniqueness of the multipliers.

The results are obtained in collaboration with Bernd Kummer (Humboldt University Berlin).

The equations of elastostatics in a Riemannian manifold

Cristinel Mardare, University Pierre et Marie Curie, France

Abstract. We will discuss the equations of elastostatics in a Riemannian manifold, which generalize those of classical elasticity in the three-dimensional Euclidean space. Our approach relies on the principle of least energy, which asserts that the deformation of the elastic body arising in response to given loads minimizes over a specific set of admissible deformations the total energy of the elastic body, defined as the difference between the strain energy and the potential of the loads. Assuming that the strain energy is a function of the metric tensor field induced by the deformation, we first derive the principle of virtual work and the associated nonlinear boundary value problem of nonlinear elasticity from the expression of the total energy of the elastic body. We then show that this boundary value problem possesses a solution if the loads are sufficiently small (in a sense we specify).

Asymptotic analysis of the Bingham flow in periodic domains

Sorin Mardare, Université de Rouen, France

Abstract. We study the flow of a Bingham fluid in a domain which is periodic in one direction. We are interested in the asymptotic behavior of the solution to the stationary Bingham problem as the length of the domain (in the periodic direction) goes to infinity. Our study is based on the techniques developed by Chipot and his collaborators for studying partial differential equations in cylindrical domains becoming unbounded. We show in particular how these techniques can be adapted in order to overcome the difficulties arising from the non-linearity of the equation. The main result is that the velocity part of the solution to the stationary Bingham problem strongly converges in the H^1 norm to the solution of another Bingham problem, this time defined in the infinite periodic domain. This result is similar to an earlier result of Chipot and M. concerning the Stokes problem, but with a lower rate of convergence. More specifically, we show that the difference between the solution to the Bingham problem in the periodic domain of length $2l$ (in the periodic direction) and its limit is of order $1/l^a$, with $0 < a < 1/2$, instead of $1/e^{al}$, for some positive constant a, in the case of the Stokes problem.

This is a joint work with Patrizia Donato and Bogdan Vernescu.

Asymptotic behavior of variants of the Cahn–Hilliard equation

Alain Miranville, University of Poitiers, France

Abstract. Our aim in this talk is to discuss the qualitative behavior (existence of finite-dimensional attractors and blow up in finite time) of variants of the Cahn–Hilliard equation. Such equations arise in the context of image inpainting and biology.

Fractional equations with critical nonlinearities

Giovanni Molica Bisci, University Mediterranea of Reggio Calabria, Italy

Abstract. n this talk we consider the following critical nonlocal problem

$$\begin{cases} -\mathcal{L}_K u = \lambda u + |u|^{2^*-2}u & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega\,, \end{cases}$$

where $s \in (0,1)$, Ω is an open bounded subset of $\mathbb{R}^n$, $n > 2s$, with Lipschitz boundary, λ is a positive real parameter, $2^* := 2n/(n-2s)$ is the fractional critical Sobolev exponent, while $\mathcal{L}_K$ is the nonlocal integrodifferential operator

$$\mathcal{L}_K u(x) := \int_{\mathbb{R}^n} \Big(u(x+y) + u(x-y) - 2u(x) \Big) K(y)\, dy\,, \quad x \in \mathbb{R}^n\,,$$

whose model is given by the fractional Laplacian $-(-\Delta)^s$.

We prove a multiplicity and bifurcation result for this problem, using a classical theorem in critical points theory. Precisely, we show that in a suitable left neighborhood of any eigenvalue of $-\mathcal{L}_K$ (with Dirichlet boundary data) the number of nontrivial solutions for the problem under consideration is at least twice the multiplicity of the eigenvalue. Hence, we extend a classical result got by Cerami, Fortunato and Struwe for classical elliptic equations, to the case of nonlocal fractional operators.

Gamma-convergence of peridynamics

Pablo Pedregal, University Castilla-La Mancha, Spain

Abstract. We will explore how long-range interaction energies in peridynamics models can give rise to local energies in the limit when the horizon goes down to zero. In particular, we are interested in understanding how the non-local energy density is related to the density of the limit energy functional.

Joint work with J. C. Bellido and C. Mora-Corral.

Viscous incompressible free-surface flow down an inclined perturbed plane

Konstantin Pileckas, Vilnius University, Lithuania

Abstract. The plane stationary free boundary value problem for the Navier–Stokes equations is studied. This problem models the viscous fluid free-surface flow down a perturbed inclined plane. For sufficiently small data the solvability and uniqueness results are proved in Hölder spaces. The asymptotic behavior of the solution is investigated.

The results are obtained jointly with V. A. Solonnikov.

Critical stationary Kirchhoff equations in $\mathbb{R}^N$ involving nonlocal operators

Patrizia Pucci, University of Perugia, Italy

Abstract. In this talk I present existence and multiplicity of nontrivial non-negative entire solutions of a stationary Kirchhoff eigenvalue problem, involving a general nonlocal integro-differential operator. The model under consideration involves two superlinear nonlinearities, one of which could be critical or even supercritical.

A priori estimates and existence for semilinear elliptic systems with power nonlinearities

Pavol Quittner, Comenius University, Slovakia

Abstract. We prove a priori estimates and existence of positive solutions of semilinear elliptic systems with power nonlinearities and homogeneous Dirichlet boundary conditions. In general, our problems are nonvariational and noncooperative, and our estimates are optimal in the class of very weak solutions.

Ground states of a nonlinear curl-curl problem

Wolfgang Reichel, KIT (Karlsruhe Institute of Technology), Germany

Abstract. In this talk I will report on recent joint work with Thomas Bartsch, Tomas Dohnal and Michael Plum. We are interested in ground states for the nonlinear curl-curl equation

$$\nabla \times \nabla \times U + V(x)U = \Gamma(x)|U|^{p-1}U \text{ in } \mathbb{R}^3, \quad U : \mathbb{R}^3 \to \mathbb{R}^3.$$

A basic requirement is to find scenarios, where 0 does not belong to the spectrum of the operator

$$\mathcal{L} = \nabla \times \nabla \times + V(x).$$

Under suitable assumptions on V, Γ we construct ground states both for the defocusing case ($\Gamma \leq 0$) and the focusing case ($\Gamma \geq 0$). The main tools are variational methods and the use of symmetries.

Characterization of local existence for nonlinear heat equations with initial data in L^1

James Robinson, University of Warwick, United Kingdom

Abstract. We consider the equation $u_t - \Delta u = f(u)$ on a bounded domain $\Omega \subset R^d$ with Dirichlet boundary conditions, under the assumption that $f : [0,\infty) \to [0,\infty)$ is continuous and non-decreasing. We show that the equation has a local solution bounded in $L^1(\Omega)$ for all initial values $u_0 \in L^1(\Omega)$ if and only if

$$\int_1^\infty s^{-(1+2/d)} F(s) \, \text{\c{s}} < \infty,$$

where $F(s) = \sup_{1 \le t \le s} f(s)/s$.

This is joint work with Robert Laister (University of the West of England), Mikolaj Sierzega (Warwick), and Alejandro Vidal-Lpez (Xian Jiaotong-Liverpool University).

Asymptotic Analysis of Eigenvalue Problems with Mixed Boundary type Data

Prosenjit Roy, Tata Institute of Fundamental Research, India

Abstract. We will consider eigenvalue problems with mixed (Dirichlet on some part and Neumann on the remaining part of the boundary) boundary type conditions. The domains (dimensions > 1) under consideration will be of cylindrical types, which will tend to become unbounded in one direction. We will study the asymptotic behavior of the eigenmodes of such problems. I will show how these problems are connected to “Problems of Dimension Reduction”.

This is joint work with Michel Chipot and Itai Shafrir.

Chain Rule for approximately differentiable homeomorphisms

Carlo Sbordone, University of Naples, Italy

Abstract. A *chain rule* for an a.e. approximatively differentiable homeomorphism $f : \Omega \subset \mathbb{R}^n onto \Omega' \subset \mathbb{R}^n$ is proved. Namely, there exists a Borel set $B \subset \mathcal{R}_f^{\mathrm{ap}} = \{x \in \Omega : f$ is approximately differentiable at x and $J_f(x) \neq 0\}$ such that $|\mathcal{R}_f^{\mathrm{ap}} \setminus B| = 0$, $f(B) \subset \mathcal{R}_{f^{-1}}^{\mathrm{ap}}$ and

$$J_{f^{-1}}(f(x))J_f(x) = 1 \qquad \forall x \in B.$$

Moreover, if f^{-1} is a.e. approximately differentiable in Ω', then $f(B)$ has full measure in $\mathcal{R}_{f^{-1}}^{\mathrm{ap}}$.

In general, it is not true that $f(\mathcal{R}_f^{\mathrm{ap}}) \subset \mathcal{R}_{f^{-1}}^{\mathrm{ap}}$. Indeed, there exists a homeomorphism $f_0 : \Omega \subset \mathbb{R}^n \overset{\text{onto}}{\longrightarrow} \Omega' \subset \mathbb{R}^n$ such that f_0 is approximately differentiable at x_0 with $J_{f_0}(x_0) \neq 0$ and f_0^{-1} is not approximately differentiable at $f_0(x_0)$.

This study was initiated in [N. Fusco, G. Moscariello, C. Sbordone: *The limit of $W^{1,1}$-homeomorphisms with finite distortion*, Calc. Var., 33, (2008), 377–390.]

Some recent results for fractional Laplacian problems

Raffaella Servadei, Università della Calabria, Italy

Abstract. In the last years, nonlocal fractional equations have attracted several outstanding mathematicians and the interest towards these problems has grown more and more, not only for their intriguing analytical structure, but also in view of their applications in a wide range of contexts.

Aim of this talk will be to present some recent results for nonlocal problems driven by the fractional Laplace operator $(-\Delta)^s$, $s \in (0,1)$, which (up to normalization factors) may be defined as

$$-(-\Delta)^s u(x) = \int_{\mathbb{R}^n} \frac{u(x+y) + u(x-y) - 2u(x)}{|y|^{n+2s}}\, dy\,, \quad x \in \mathbb{R}^{\ltimes}\,.$$

Asymptotic behavior of critical points of an energy involving a “circular-well” potential

Itai Shafrir, Technion, Israel

Abstract. We study the singular limit of critical points of an energy with a penalization term depending on a small parameter. The energy involves a potential which is a nonnegative function on the plane, vanishing on a closed curve. We generalize to this setting results obtained by Bethuel, Brezis and Helein for the Ginzburg-Landau energy.

This is a joint work with Petru Mironescu (Lyon I).

Asymptotic conditions at infinity for the time-periodic Stokes problem set in domains with cylindrical outlets

Mindaugas Skujus, Vilnius University, Lithuania

Abstract. We consider the time-periodic Stokes system set in domains with several cylindrical outlets to infinity. This mathematical model describes the pulsating flow of the viscous incompressible fluid in systems of thin and long pipes. We present the generalized Green formula, which allows us to formulate so called asymptotic conditions at infinity. These conditions ensure the unique solvability of the Stokes problem in the class of functions with unbounded Dirichlet's integrals. We present also the special class of asymptotic conditions at infinity which models flows with prescribed flow-rates and pressures in pipes.

Finite-time blowup and global-in-time unbounded solutions to a parabolic-parabolic Keller–Segel system

Christian Stinner, University of Kaiserslautern, Germany

Abstract. Several variants of the Keller-Segel model are used in mathematical biology to describe the evolution of cell populations due to both diffusion and chemotactic movement. We study radially symmetric solutions to a quasilinear parabolic-parabolic Keller-Segel system in a ball in $\mathbb{R}^n$ for $n \geq 2$. Critical nonlinearities had been identified such that in the subcritical case the solution is global in time and bounded while in the supercritical case the solution blows up, but it was not known whether the blowup takes place in finite or infinite time. Assuming a condition on the growth of the chemotactic sensitivity function, we prove that any solution blows up in finite time in the whole supercritical case. Moreover, we provide an example showing that in presence of a suitable decay of the sensitivity function some solutions blow up in infinite time in the supercritical case without any restriction concerning the initial mass. An important ingredient of our proof is a detailed analysis of a Liapunov functional.

This is a joint work with T. Cieslak (Warsaw).

Weighted estimates for the solutions of Dirichlet problems in irregular domains

Maria Agostina Vivaldi, Sapienza Università di Roma, Italy

Abstract. In this talk we consider a sequence pre-fractal domains approximating the Koch snowflake. These pre-fractal domains are polygonal, non convex and with an increasing number of sides. More precisely, the boundary is a polygonal curve developing at the limit a fractal geometry. As consequence, it is the union of an increasing number of graphs. The (classical) regularity results, in weighted Sobolev spaces, for the solutions of the PDEs provide estimates that involve constants that diverge as the number of graphs become infinity. In this talk we discuss uniform estimates, in weighted Sobolev spaces, for the solutions of the PDEs on the pre-fractal domains as well as regularity results (in weighted Sobolev spaces) for the solutions of the PDEs on the snowflake domain. Our proof combines the pioneer results of Kondratiev with the sophisticated techniques introduced for the NTA domains.

Finite-time blowup for a complex Ginzburg–Landau equation

Frederic Weissler, Université Paris 13, Sorbonne Paris Cité, France

Abstract. We prove that negative energy solutions of the complex Ginzburg-Landau equation $e^{-i\theta}u_t = \Delta u + |u|^\alpha u$ blow up in finite time, where $\alpha > 0$ and $-\pi/2 < \theta < \pi/2$. For a fixed initial value $u(0)$, we obtain estimates of the blow-up time T^θ_{max} as $\theta \to \pm\pi/2$. It turns out that T^θ_{max} stays bounded (respectively, goes to infinity) as $\theta \to \pm\pi/2$ in the case where the solution of the limiting nonlinear Schrödinger equation blows up in finite time (respectively, is global).

Joint work with : T. Cazenave and F. Dictstein

Spreading and vanishing dichotomy for some free boundary problems in population biology

Yoshio Yamada, Waseda University, Japan

Abstract. This talk is concerned with some free boundary problems which model the invasion or migration of a biological species. The species lives in a bounded habitat and its boundary (or a part of its boundary) is a moving free boundary. In this model, the population density is described by a certain class of reaction diffusion equations and the dynamics of the moving free boundary is controlled by a Stefan-like condition. We will show a spreading and vanishing dichotomy result for asymptotic behavior of solutions. Moreover, we will also give some criteria for spreading or vanishing of solutions.

Nondegeneracy in the tick obstacle problem

Karen Yeressian, University of Zurich, Switzerland

Abstract (latex): In this talk I will present the optimal nondegeneracy of the solution u of the obstacle problem

$$\triangle u = f\chi_{\{u>0\}}$$

in a bounded domain $D \subset \mathbb{R}^n$, where the only requirement on f is that it should have a nondegeneracy of the type $\lambda|(x_1, \cdots, x_p)|^\alpha \leq f(x)$ for some $\lambda > 0$, $1 \leq p \leq n$ (an integer) and $\alpha > 0$. I will present the proof of the optimal uniform $(2+\alpha)$-th order and nonuniform quadratic nondegeneracy. Also I bring the proof of the optimal growth with the assumption $|f(x)| \leq \Lambda|(x_1, \cdots, x_p)|^\alpha$ for some $\Lambda \geq 0$ and the proof of the porosity of the free boundary.