Interior point method for time-dependent inverse problems

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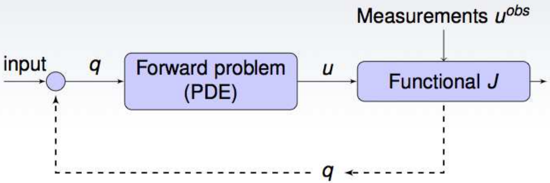
Motivation



Seismic exploration

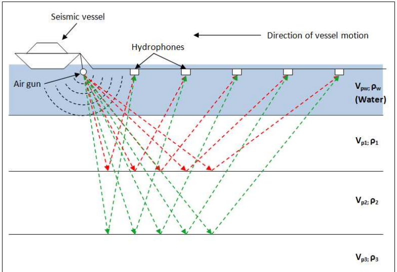


Diagram of a marine towed streamer seismic survey with the raypaths that result from a single shot by an airgun into a streamer containing 5 hydrophones.
--- = raypaths to first reflector; --- = raypaths to second reflector;

We have to define:

- the *PDE*
- the functional J
- the variable q

[Illustration from www.earthscrust.org]

Outline

1. PDE Constrained Optimization Problem

2. Numerical Results

PDE Constrained Optimization Problem

Problem

Let $\Omega \subset \mathbb{R}^2$ be a bounded open set with a Lipschitz boundary $\partial\Omega$, we seek $(u, q) \in V \times Q_{ad}$:

$$\begin{cases} \min\limits_{(u,q)} J(u, q) \\ \text{sbj to } \mathcal{A}(u, q) = 0 \end{cases}$$

being

- V, Q Hilbert spaces
- a control variable $q \in Q_{ad} \subseteq Q$
- a state variable $u \in V$
- a forward or state problem $\mathcal{A}(u, q) = 0$
- an misfit functional $J = J(u, q)$

First discretize then optimize

Space-time discretization

- in space: Finite Difference, Finite Element, Discontinuous Galerkin, Spectral Element methods
- in time : Adams Bashforth, Runge Kutta, Leap Frog methods

Line search methods for Constrained optimization

- method: Active-set, Interior Point methods
- algorithm: Steepest Descent, Nonlinear Coniugate Gradient, Gauss -Newton, Newton methods

1d - model case: fr=3

Input

- $\Omega =]x_a, x_b[= (0,1),\ \Gamma_N = \partial\Omega$
- $\Omega_o = x_a, x_b$
- $Ne = 30,\ N_k = N = 2\ \forall k$
- $\alpha = 1e-9,\ tol = 1e-7,$
- $f = \sum_{j=1}^{2} f_t(t)\delta(x - x_{r_j})$

$$f_t(t) = -10(1 - 4\tau(t)^2(t))\exp(-2\tau(t)^2(t))$$

$$\tau(t) = \frac{\pi(t - fr^{-1})}{1.5 fr^{-1}}$$

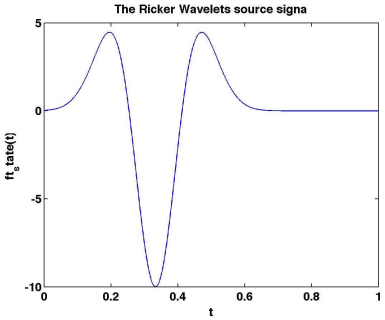
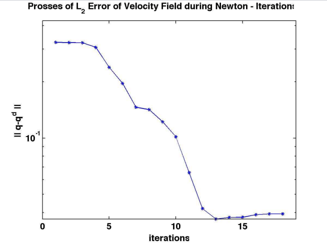
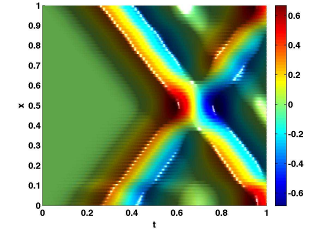


Figure: Source signal

$$J(y,q) = \frac{1}{2}\int_0^T \sum_{i=1}^{2} (u - z_d)^2\, d\Omega\, dt + \frac{\alpha}{2} R(q).$$

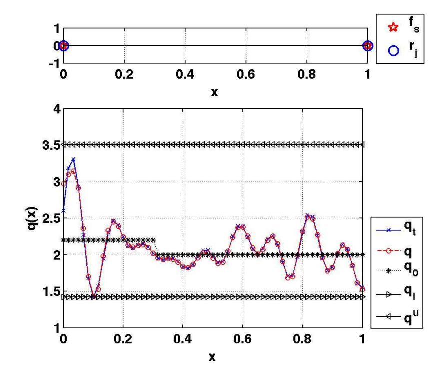


(a) Gradient norm

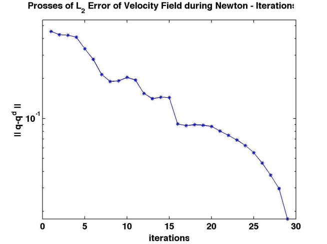


(b) Forward wave simulation

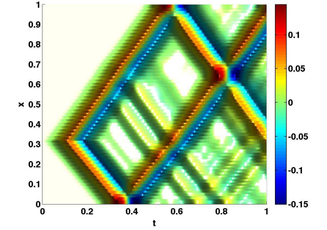
2 sources and 2 receivers!



(c) Velocity fields

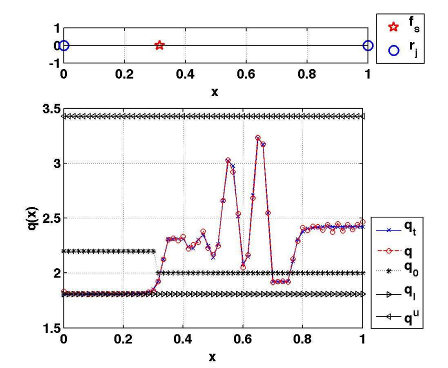


(d) Gradient norm



(e) Forward wave simulation

active contraints! 1 sources and 2 receivers!



(f) Velocity fields

Concluding Remarks

Functional: full-waveform inversion
PDE: second-order wave equation

Forward problem: LF-SEM discretization

- + : diagonal mass-matrix
- + : accuracy

- ✓ : different obsersations and sources location
- ✗ : stability restriction → Local Time Stepping Methods

Optimization: Ipopt method

- ✓ : control contrains
- ✗ : state constraints
- ✗ : extension to higher dimension

Thank you for your attention!

Unpublished work, please contact L.Gaudio for additional details.