Multiobjective approximation models arising in radiotherapy treatment in medicine and healthcare logistics

Part 3: Multiobjective Optimization, programming

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1. Algorithm for generating the solution set to multiobjective location problems
2. Software Facility Location Optimizer (FLO)
3. Conclusions

1. Algorithm for generating the whole set of solutions to multiobjective location problems

Example for multiobjective location problems arising in healthcare logistics in Part 1: Location for a new Vaccination Center in a district of the town Havana:

Consider a certain district of the town Havana with m existing facilities. The decision makers in the public health department of Havana are looking for a location $x \in \mathbb{R}^2$ for a new Vaccination Center such that the distances between the existing facilities $a^1, \ldots, a^m \in \mathbb{R}^2$ and the location for the new Vaccination Center $x \in \mathbb{R}^2$ are to be minimized in the sense of multiobjective optimization. The distances are measured by a norm $\| \cdot \|$ in $\mathbb{R}^2$. So, we study the following multiobjective location problem:

$$f(x) = \begin{pmatrix} \|x - a^1\| \\ \|x - a^2\| \\ \cdots \\ \|x - a^m\| \end{pmatrix} \longrightarrow \min_{x \in \mathbb{R}^2} \quad \text{w.r.t. } \mathbb{R}^m_+. \qquad \text{(POLP)}$$

Remark 1. *(POLP) means that we are looking for the set* $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$*. For applications in town planning it is important that we can choose different norms in the formulation of (POLP). The decision which of the norms will be used depends on the course of the roads in the city or in the district. In the following, we consider (POLP) where the maximum norm is involved*

$$\|x\|_{\max} = \max\{| x_1 |, | x_2 |\}.$$

Using duality assertions it is possible to derive an algorithm for solving (POLP) (compare Chalmet, Francis, and Kolen (1981), Gerth and Pöhler (1988)).

The dual problem to (POLP):

$$\text{Determine} \quad \mathrm{Eff}_{\mathrm{Max}}((\mathbb{R}^2)^m \mid f^*) \tag{D}$$

where $f^*(Y) := \begin{pmatrix} Y^1(a^1) \\ \cdots \\ Y^n(a^m) \end{pmatrix}$ and

$$\mathcal{B} = \{Y = (Y^1, \ldots, Y^m), Y^i \in L(\mathbb{R}^2, \mathbb{R}) : \exists\, \lambda^* \in \operatorname{int} \mathbb{R}^m_+ \quad \text{with}$$

$$\sum_{i=1}^{n} \lambda_i^* Y^i = 0, \quad \text{and} \quad \|Y^i\|_1 \leq 1 \quad (i = 1, \ldots, m)\}.$$

Here $\| \cdot \|_*$ denotes the Manhattan norm ($\|x\|_1 := |x_1| + |x_2|$ for $x \in \mathbb{R}^2$). The conditions $\Sigma_{i=1}^m \lambda_i^* Y^i = 0$, and $\|Y^i\|_1 \leq 1 \quad (i = 1, \ldots, m)$ are used for deriving an algorithm.

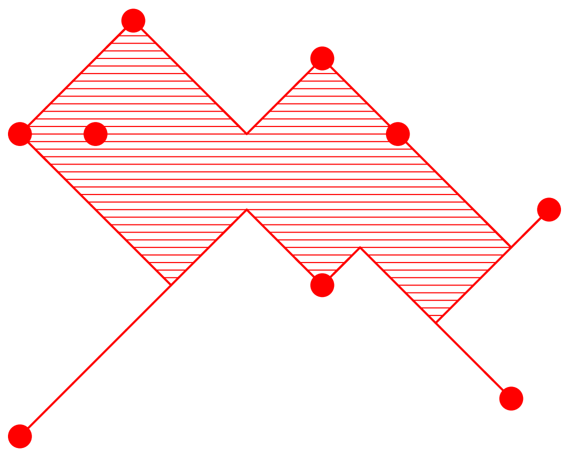
Consider the following sets with respect to $a^i \in \mathbb{R}^2$ $(i = 1, \ldots, m)$,

$$
\begin{aligned}
s_1(a^i) \quad &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = a_2^i - x_2 \geq 0\}, \\
s_2(a^i) \quad &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = a_2^i - x_2 \leq 0\}, \\
s_3(a^i) \quad &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = x_2 - a_2^i \geq 0\}, \\
s_4(a^i) \quad &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = x_2 - a_2^i \leq 0\}, \\
s_5(a^i) \quad &= \{x \in \mathbb{R}^2 \mid a_2^i - x_2 > |a_1^i - x_1|\}, \\
s_6(a^i) \quad &= \{x \in \mathbb{R}^2 \mid x_2 - a_2^i > |a_1^i - x_1|\}, \\
s_7(a^i) \quad &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 > |a_2^i - x_2|\}, \\
s_8(a^i) \quad &= \{x \in \mathbb{R}^2 \mid x_1 - a_1^i > |a_2^i - x_2|\}.
\end{aligned}
$$

Furthermore, consider $\mathcal{S}_r := \{x \in \mathcal{N} \mid \exists\, i \in \{1, \ldots, m\}$ and $x \in s_r(a^i)\}$ $(r = 5, 6, 7, 8)$, where $\mathcal{N}$ denotes the smallest level set of the dual norm to the maximum norm (Manhattan norm) containing the points a^i $(i = 1, \ldots, m)$.

Algorithm for solving (POLP) with the maximum norm:

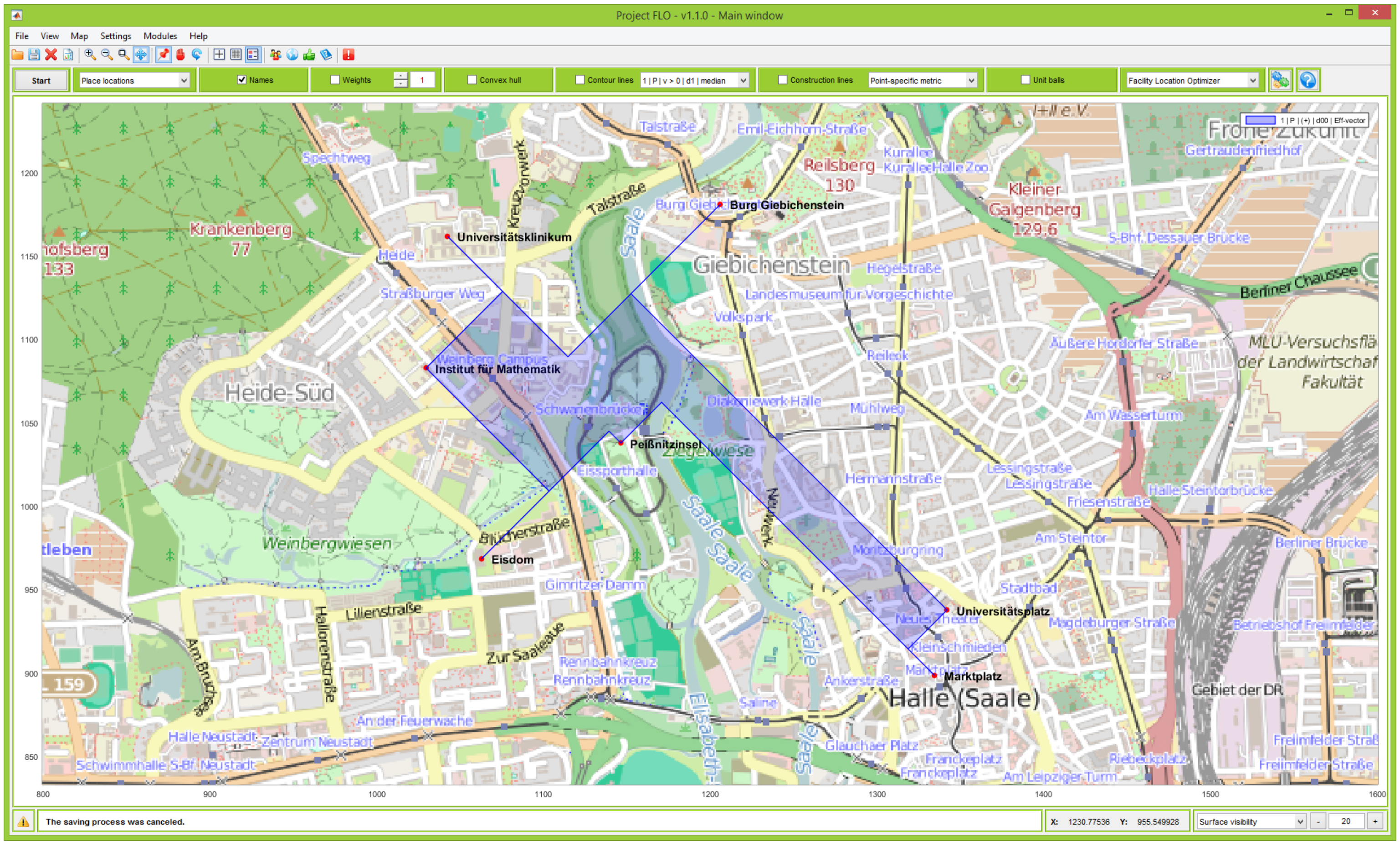
$$\begin{aligned}\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f) = &\{(\mathrm{cl}\,\mathcal{S}_5 \cap \mathrm{cl}\,\mathcal{S}_6) \cup [(\mathcal{N} \setminus \mathcal{S}_5) \cap (\mathcal{N} \setminus S_6)]\} \cap \\ &\{(\mathrm{cl}\,\mathcal{S}_7 \cap \mathrm{cl}\,\mathcal{S}_8) \cup [(\mathcal{N} \setminus \mathcal{S}_7) \cap (\mathcal{N} \setminus \mathcal{S}_8)]\}.\end{aligned}$$



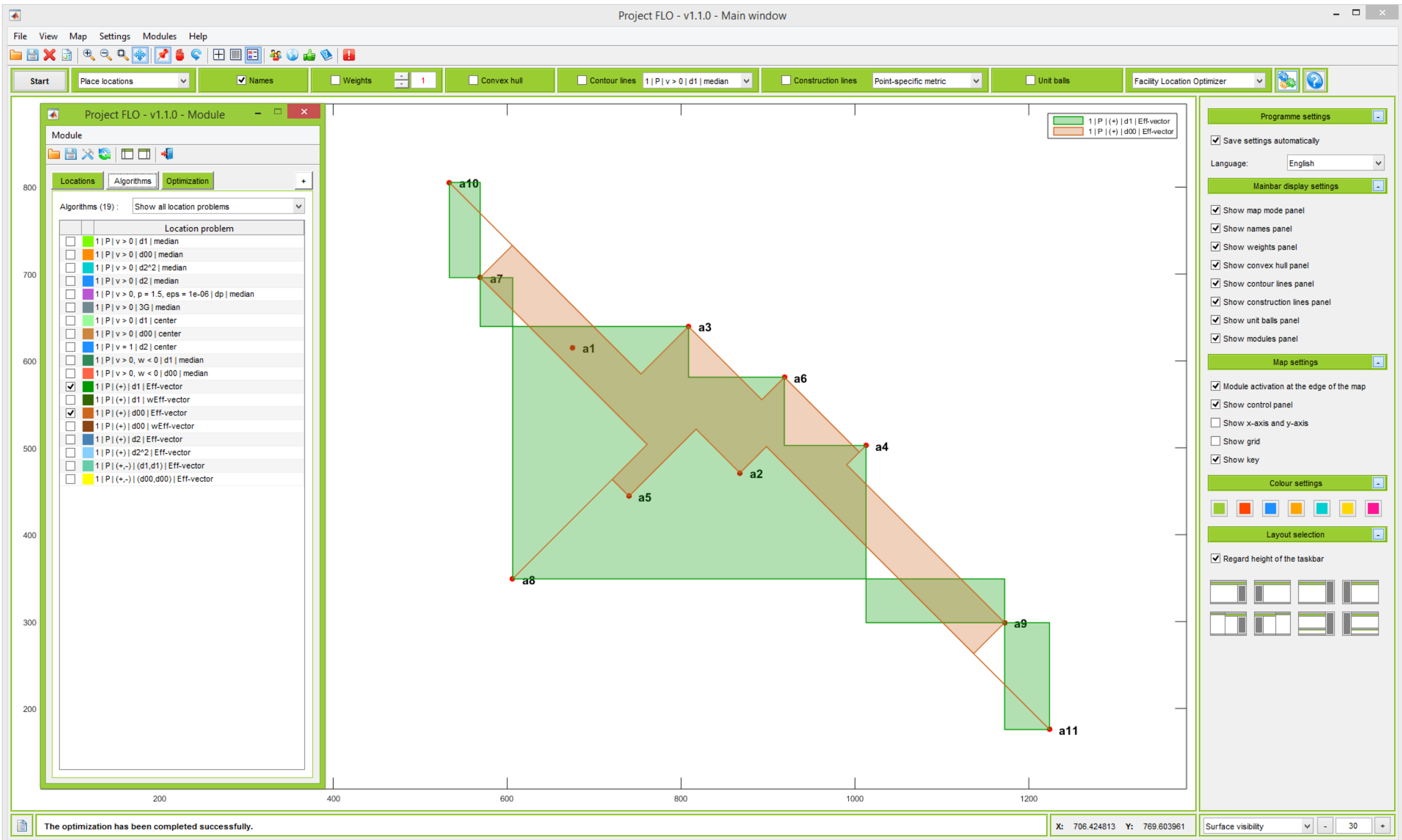
The set of efficient elements $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm.

2. Software Facility Location Optimizer (FLO)

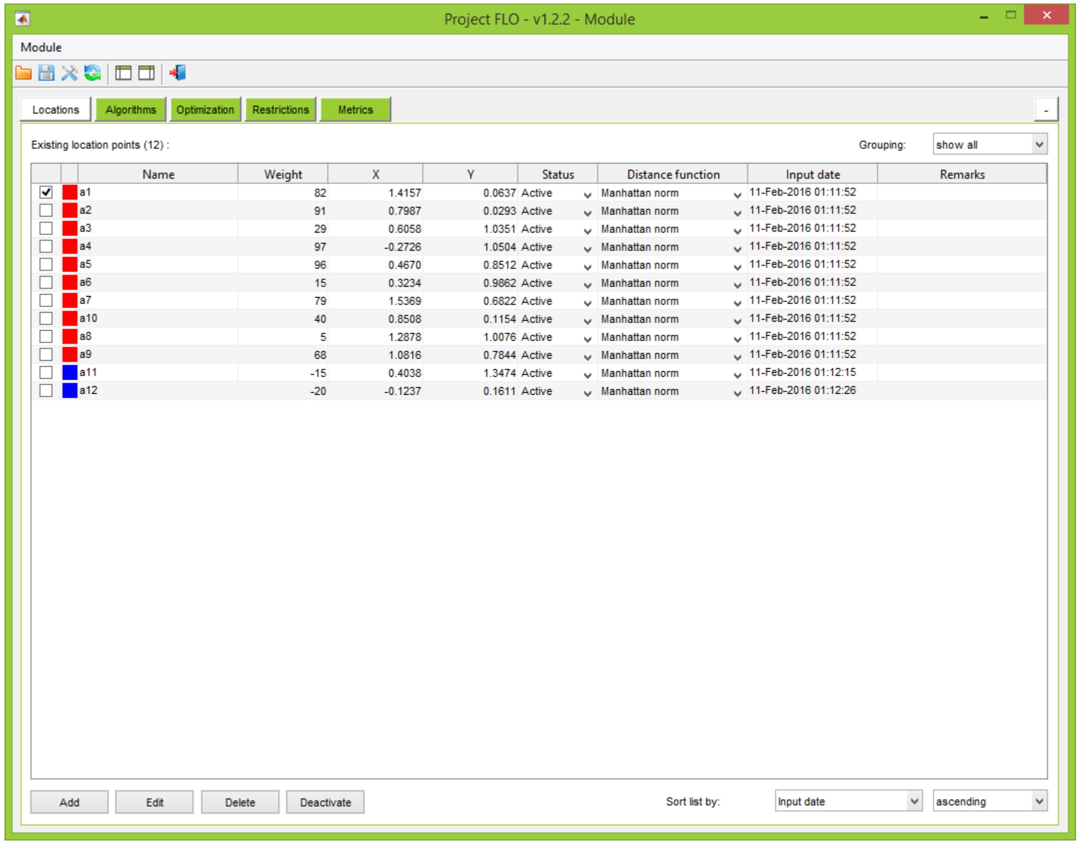
https://project-flo.de



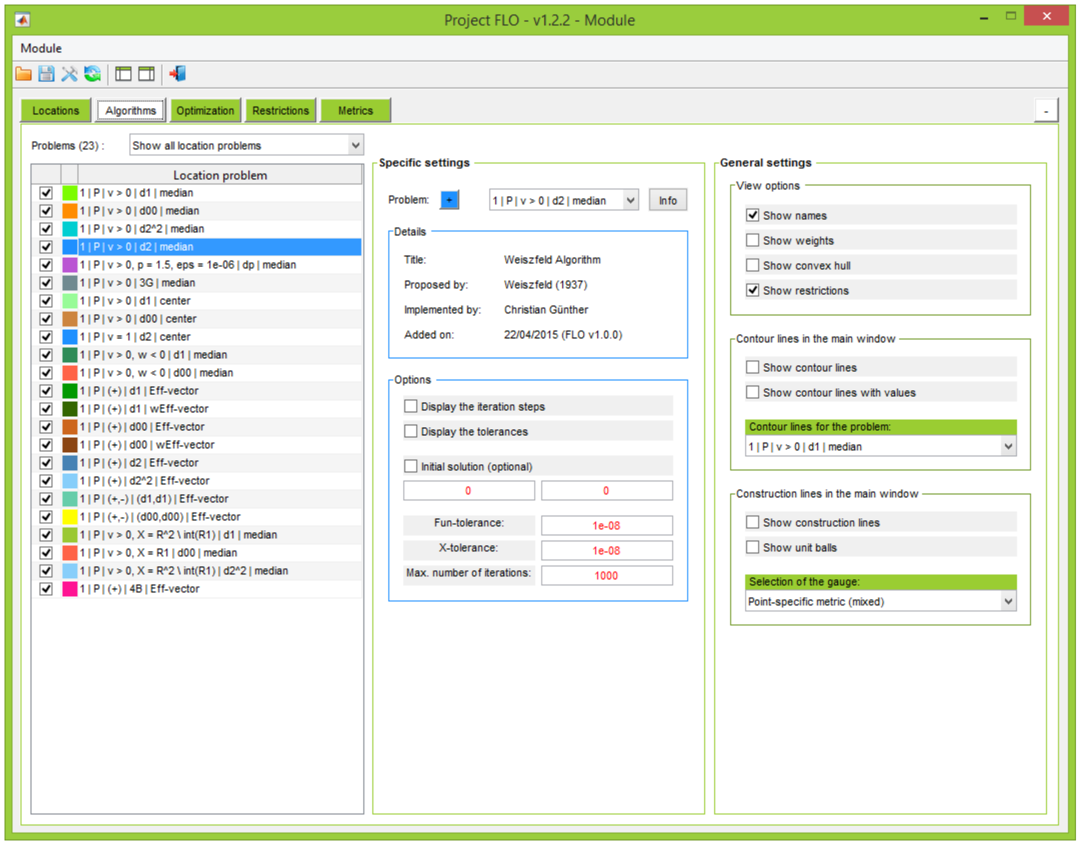
The set of efficient elements $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm.



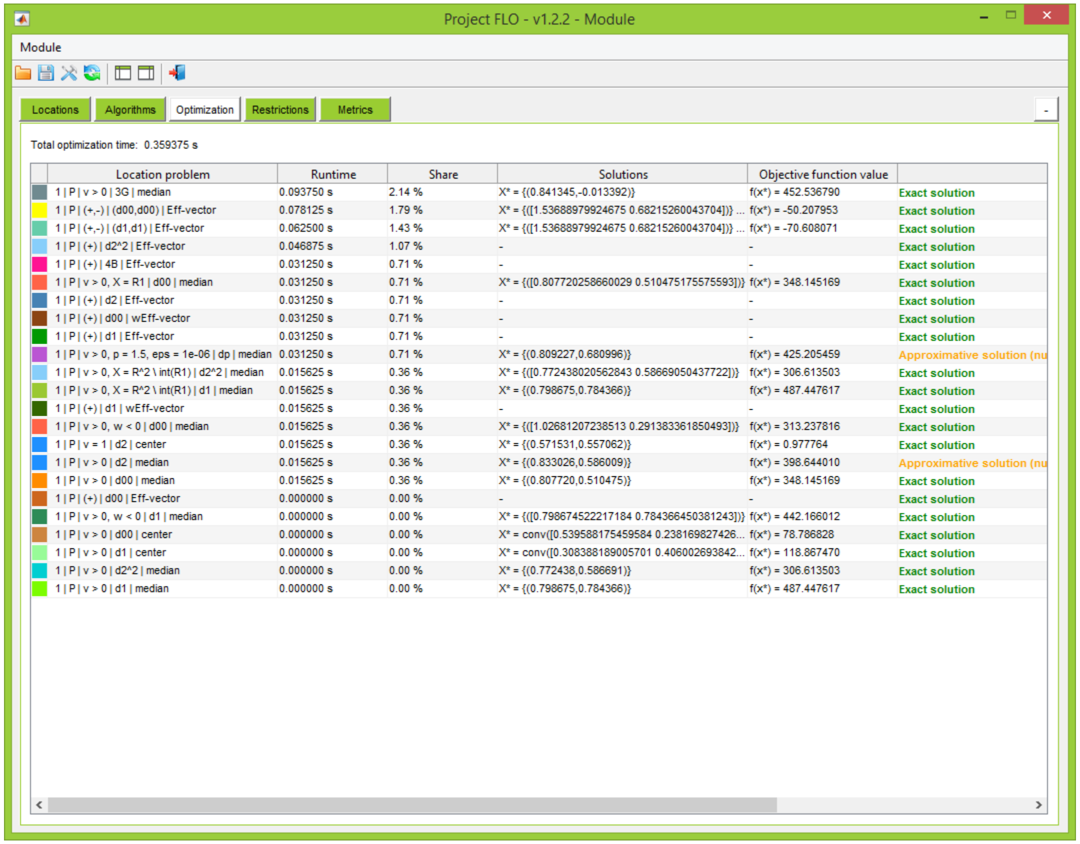
The set of efficient elements $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm (red color) and the Manhattan norm (green color).



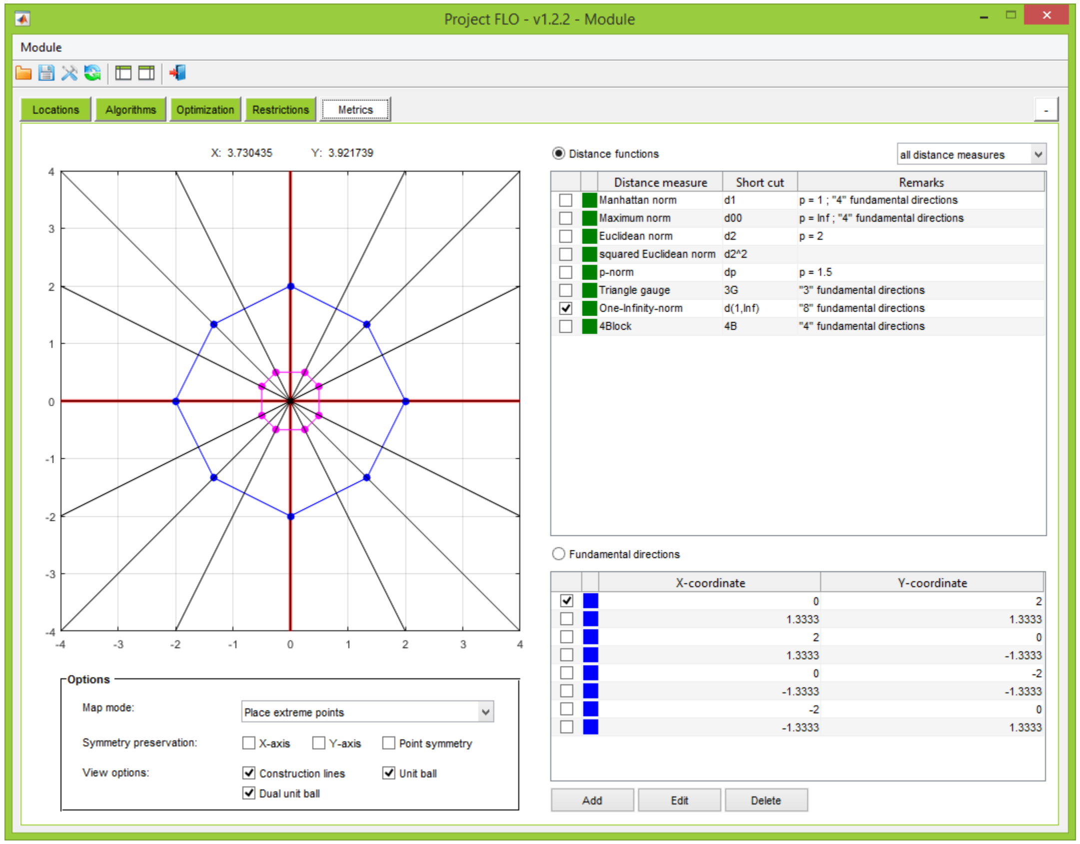
Software FLO: Locations. Entering the coordinates of the existing facilities.



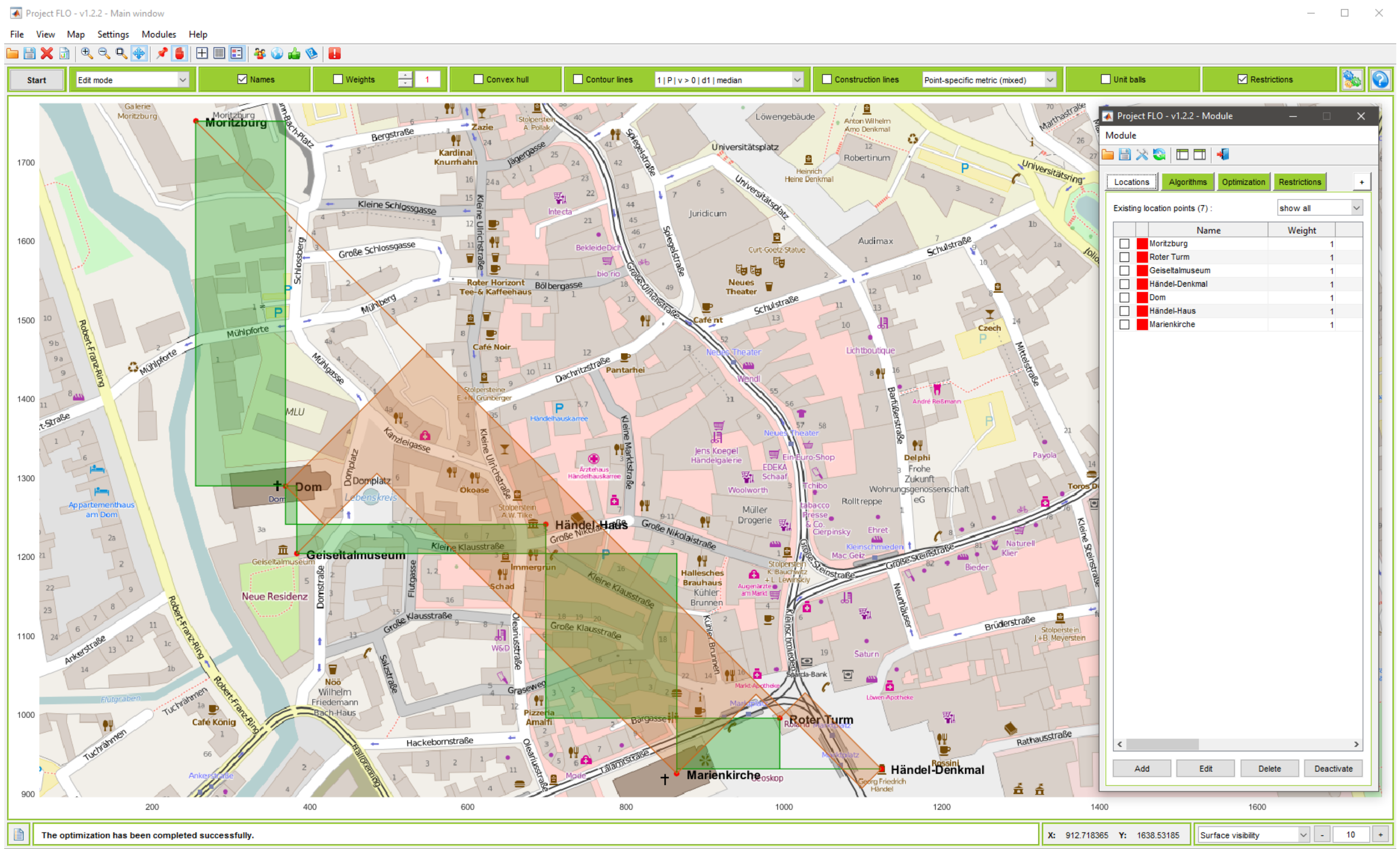
Software FLO: Algorithms. Determine the type of the location problem, especially the distance.



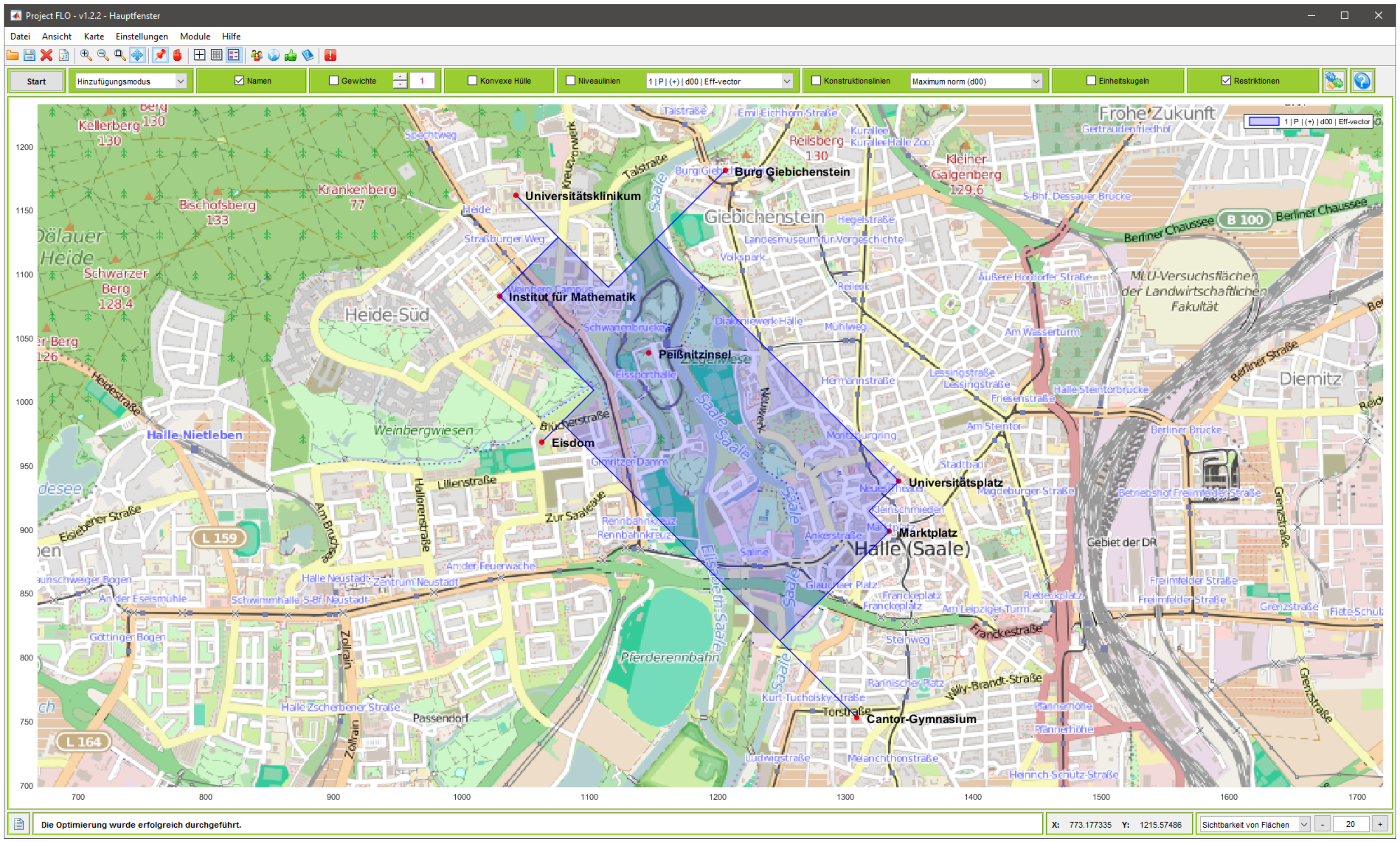
Software FLO: Optimization. Determine the solutions of the location problem using different algorithms depending from the type of the problem.



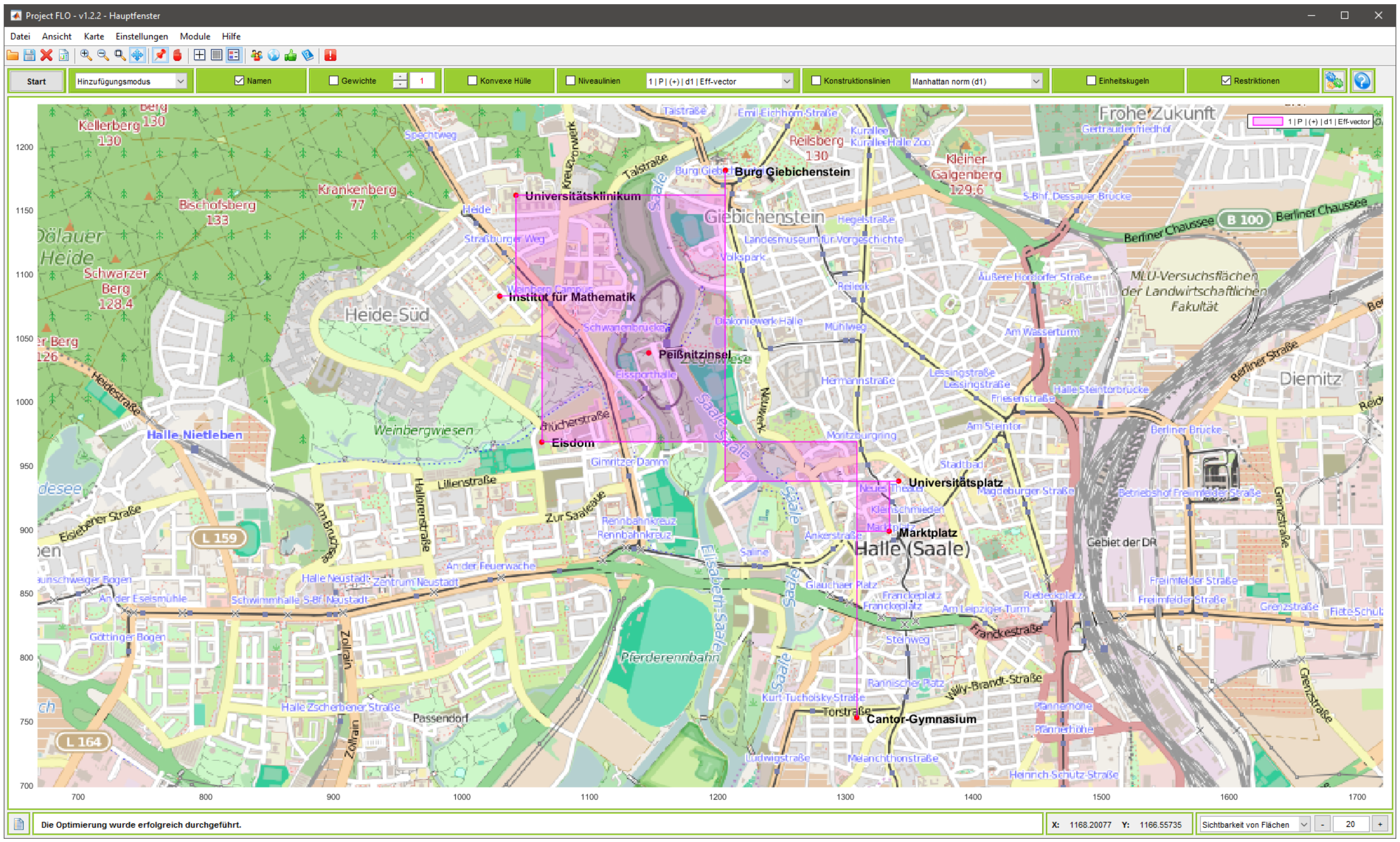
Software FLO: Metrics. Extreme points, level sets, unit balls.



Software FLO: Set of efficient elements $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$ of (POLP) with the maximum norm (red color) and with the Manhattan norm (green color) and locations.



Software FLO: The set of efficient elements $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm.



Software FLO: The set of efficient elements $\mathrm{Eff}_{\mathrm{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the Manhattan norm.

3. Conclusions

Enhancements to the software FLO:

- To include algorithms for solving constrained multiobjective location problems.

- To include algorithms for solving multiobjective approximation problems.

- To derive new and efficient algorithms for solving multiobjective location and approximation problems.

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