

Multiobjective approximation models arising in radiotherapy treatment in medicine and healthcare logistics

Part 3: Multiobjective Optimization, programming

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1. Algorithm for generating the solution set to multiobjective location problems
2. Software Facility Location Optimizer (FLO)
3. Conclusions

1. Algorithm for generating the whole set of solutions to multiobjective location problems

Example for multiobjective location problems arising in healthcare logistics in Part 1: Location for a new **Vaccination Center** in a district of the town Havana:

Consider a **certain district of the town Havana** with m existing facilities. The decision makers in the public health department of Havana are looking for a location $x \in \mathbb{R}^2$ for a new **Vaccination Center** such that the distances between the existing facilities $a^1, \dots, a^m \in \mathbb{R}^2$ and the location for the new Vaccination Center $x \in \mathbb{R}^2$ are to be **minimized** in the sense of multiobjective optimization. The distances are measured by a norm $\| \cdot \|$ in \mathbb{R}^2 . So, we study the following **multiobjective location problem**:

$$f(x) = \begin{pmatrix} \|x - a^1\| \\ \|x - a^2\| \\ \dots \\ \|x - a^m\| \end{pmatrix} \longrightarrow \min_{x \in \mathbb{R}^2} \quad \text{w.r.t. } \mathbb{R}_+^m. \quad (\text{POLP})$$

Remark 1. (POLP) means that we are looking for the set $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$. For applications in town planning it is important that we can choose *different norms* in the formulation of (POLP). The decision which of the norms will be used depends on the *course of the roads in the city or in the district*. In the following, we consider (POLP) where the *maximum norm* is involved

$$\|x\|_{\max} = \max\{|x_1|, |x_2|\}.$$

Using *duality assertions* it is possible to derive an algorithm for solving (POLP) (compare Chalmet, Francis, and Kolen (1981), Gerth and Pöhler (1988)).

The **dual problem** to (POLP):

$$\text{Determine } \text{Eff}_{\text{Max}}((\mathbb{R}^2)^m \mid f^*) \quad (\text{D})$$

$$\text{where } f^*(Y) := \begin{pmatrix} Y^1(a^1) \\ \cdots \\ Y^n(a^m) \end{pmatrix} \text{ and}$$

$$\mathcal{B} = \{Y = (Y^1, \dots, Y^m), Y^i \in L(\mathbb{R}^2, \mathbb{R}) : \exists \lambda^* \in \text{int } \mathbb{R}_+^m \text{ with}$$

$$\sum_{i=1}^m \lambda_i^* Y^i = 0, \text{ and } \|Y^i\|_1 \leq 1 \quad (i = 1, \dots, m)\}.$$

Here $\|\cdot\|_*$ denotes the **Manhattan norm** ($\|x\|_1 := |x_1| + |x_2|$ for $x \in \mathbb{R}^2$). The conditions $\sum_{i=1}^m \lambda_i^* Y^i = 0$, and $\|Y^i\|_1 \leq 1 \quad (i = 1, \dots, m)$ are used for deriving an algorithm.

Consider the following sets with respect to $a^i \in \mathbb{R}^2$ ($i = 1, \dots, m$),

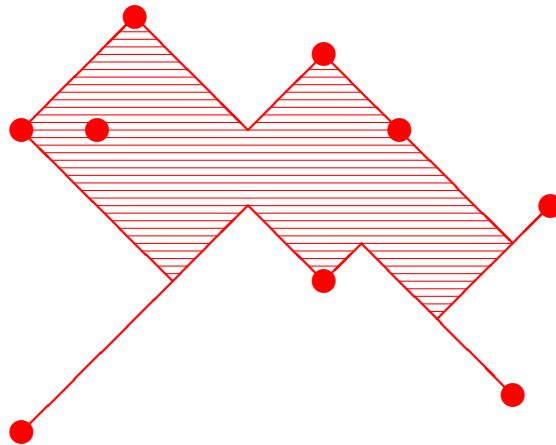
$$\begin{aligned}
 s_1(a^i) &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = a_2^i - x_2 \geq 0\}, \\
 s_2(a^i) &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = a_2^i - x_2 \leq 0\}, \\
 s_3(a^i) &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = x_2 - a_2^i \geq 0\}, \\
 s_4(a^i) &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 = x_2 - a_2^i \leq 0\}, \\
 s_5(a^i) &= \{x \in \mathbb{R}^2 \mid a_2^i - x_2 > |a_1^i - x_1|\}, \\
 s_6(a^i) &= \{x \in \mathbb{R}^2 \mid x_2 - a_2^i > |a_1^i - x_1|\}, \\
 s_7(a^i) &= \{x \in \mathbb{R}^2 \mid a_1^i - x_1 > |a_2^i - x_2|\}, \\
 s_8(a^i) &= \{x \in \mathbb{R}^2 \mid x_1 - a_1^i > |a_2^i - x_2|\}.
 \end{aligned}$$

Furthermore, consider $\mathcal{S}_r := \{x \in \mathcal{N} \mid \exists i \in \{1, \dots, m\} \text{ and } x \in s_r(a^i)\}$

($r = 5, 6, 7, 8$), where \mathcal{N} denotes the smallest level set of the dual norm to the maximum norm (Manhattan norm) containing the points a^i ($i = 1, \dots, m$).

Algorithm for solving (POLP) with the maximum norm:

$$\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f) = \{(\text{cl } \mathcal{S}_5 \cap \text{cl } \mathcal{S}_6) \cup [(\mathcal{N} \setminus \mathcal{S}_5) \cap (\mathcal{N} \setminus \mathcal{S}_6)]\} \cap \{(\text{cl } \mathcal{S}_7 \cap \text{cl } \mathcal{S}_8) \cup [(\mathcal{N} \setminus \mathcal{S}_7) \cap (\mathcal{N} \setminus \mathcal{S}_8)]\}.$$

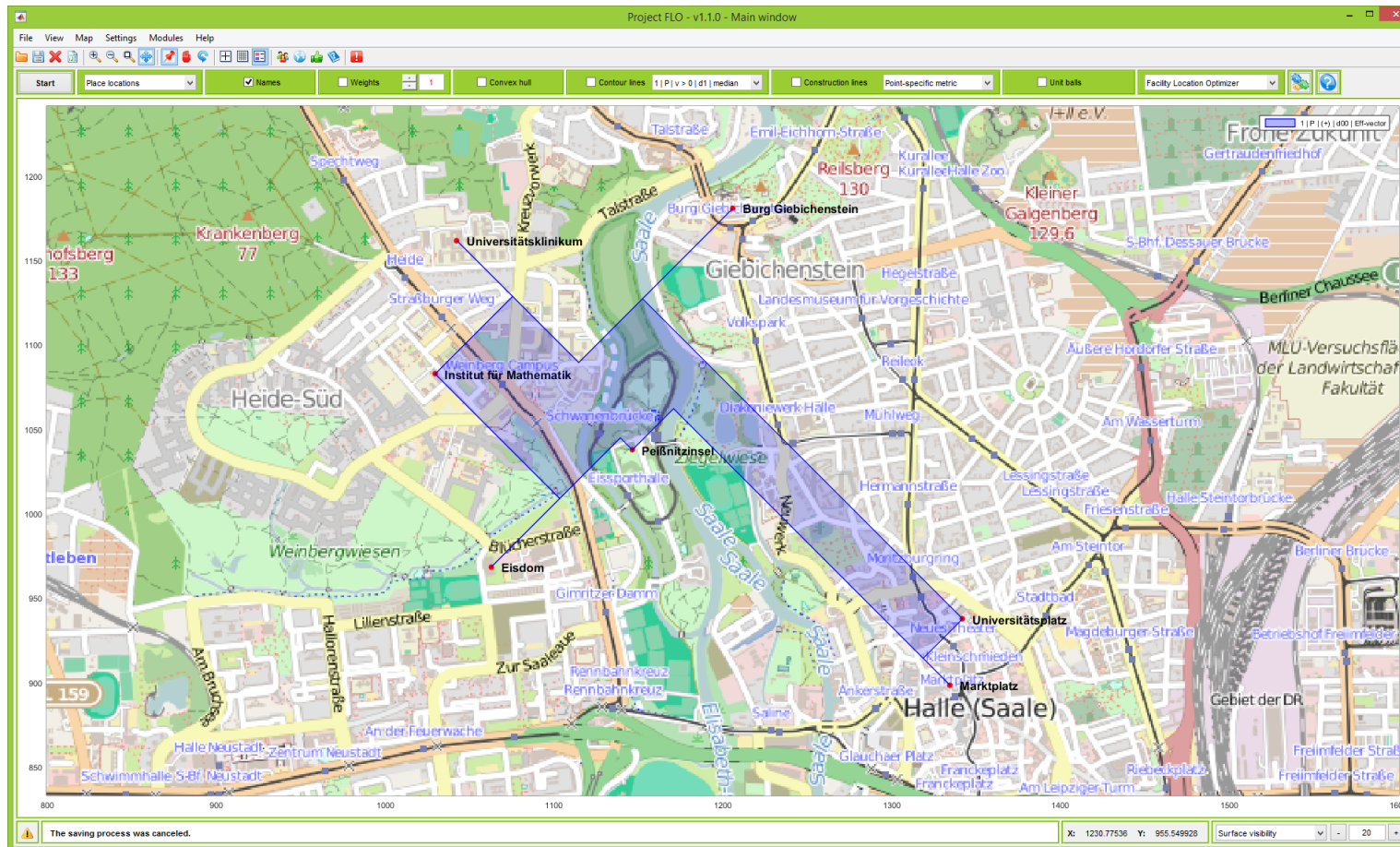


The set of efficient elements $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm.

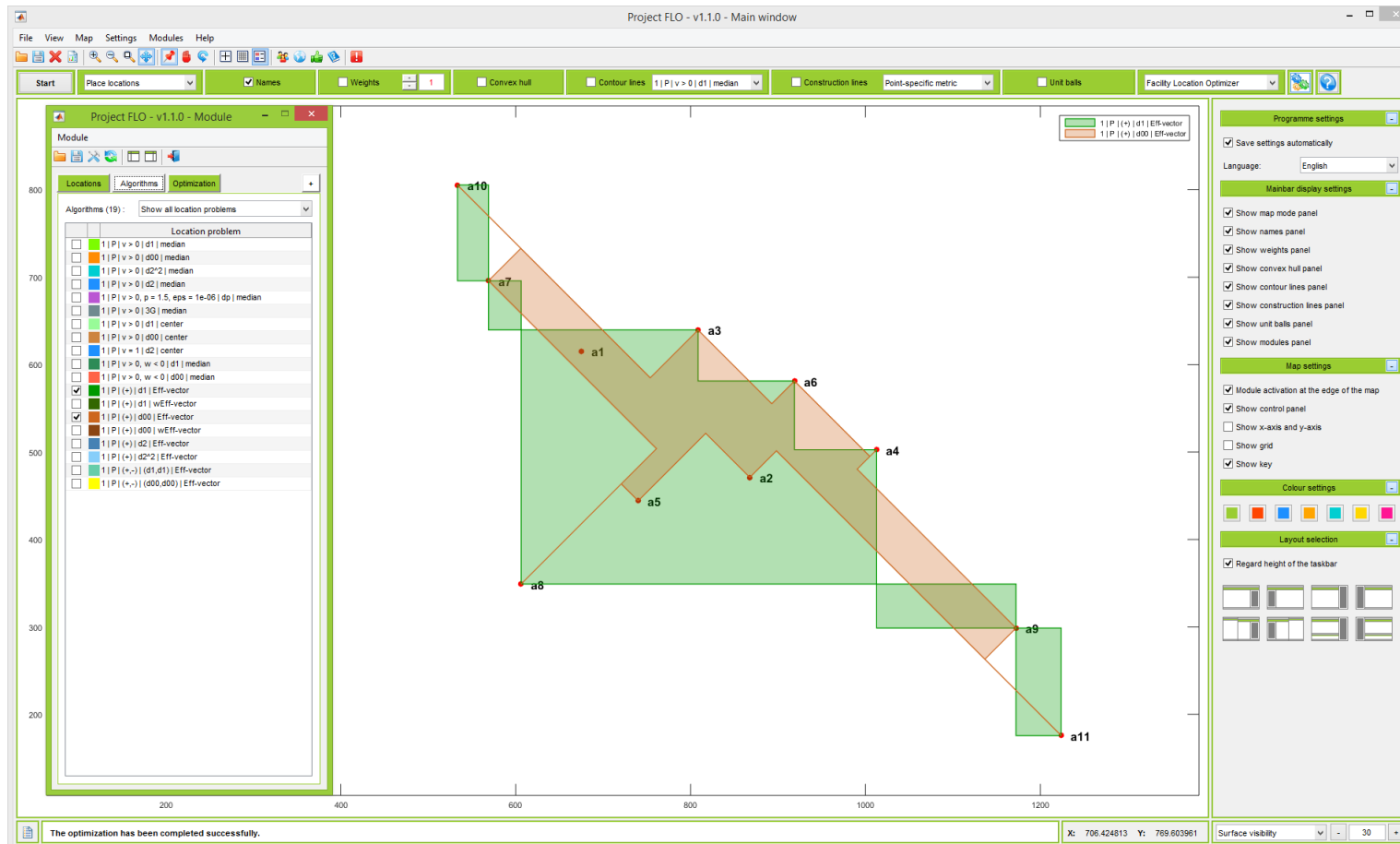
2. Software Facility Location Optimizer (FLO)



<https://project-flo.de>



The set of efficient elements $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm.



The set of efficient elements $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm (red color) and the Manhattan norm (green color).

Project FLO - v1.2.2 - Module

Module

Locations Algorithms Optimization Restrictions Metrics

Existing location points (12):

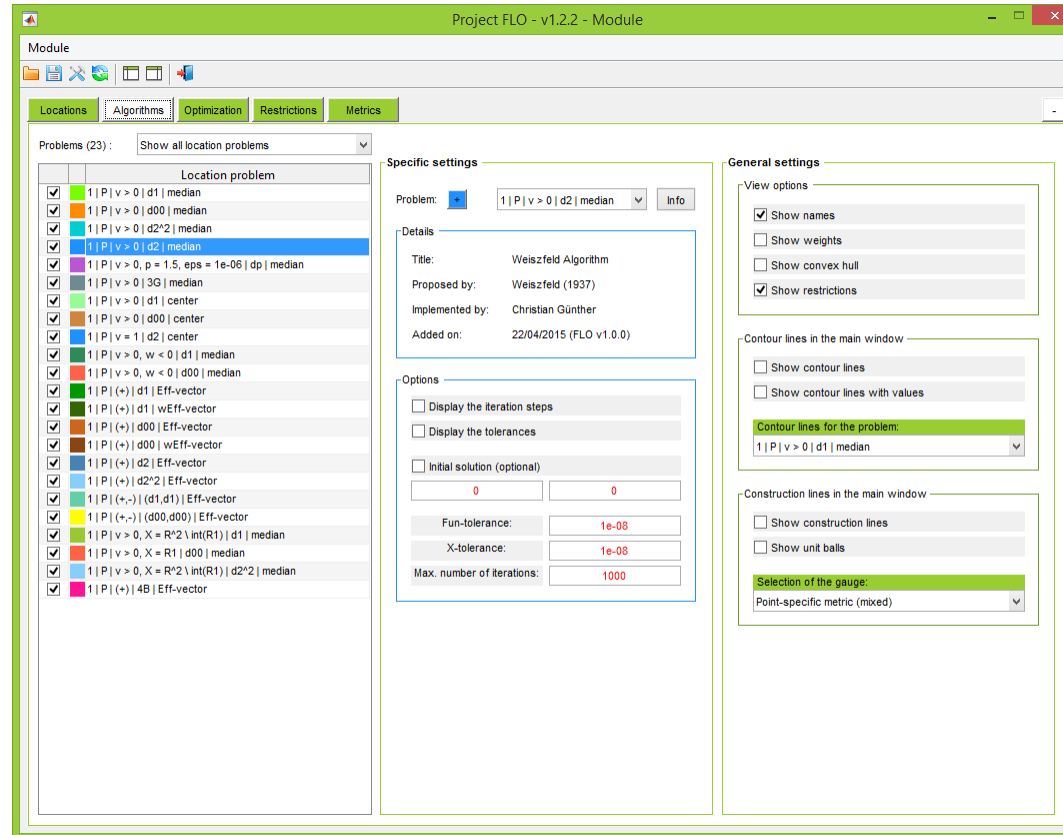
Grouping: show all

	Name	Weight	X	Y	Status	Distance function	Input date	Remarks
<input checked="" type="checkbox"/>	a1	82	1.4157	0.0637	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a2	91	0.7987	0.0293	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a3	29	0.6058	1.0351	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a4	97	-0.2726	1.0504	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a5	96	0.4670	0.8512	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a6	15	0.3234	0.9862	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a7	79	1.5369	0.6822	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a10	40	0.8508	0.1154	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a8	5	1.2878	1.0076	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a9	68	1.0816	0.7844	Active	Manhattan norm	11-Feb-2016 01:11:52	
<input type="checkbox"/>	a11	-15	0.4038	1.3474	Active	Manhattan norm	11-Feb-2016 01:12:15	
<input type="checkbox"/>	a12	-20	-0.1237	0.1611	Active	Manhattan norm	11-Feb-2016 01:12:26	

Add Edit Delete Deactivate

Sort list by: input date ascending

Software FLO: Locations. Entering the coordinates of the existing facilities.



Software FLO: Algorithms. Determine the type of the location problem, especially the distance.

Project FLO - v1.2.2 - Module

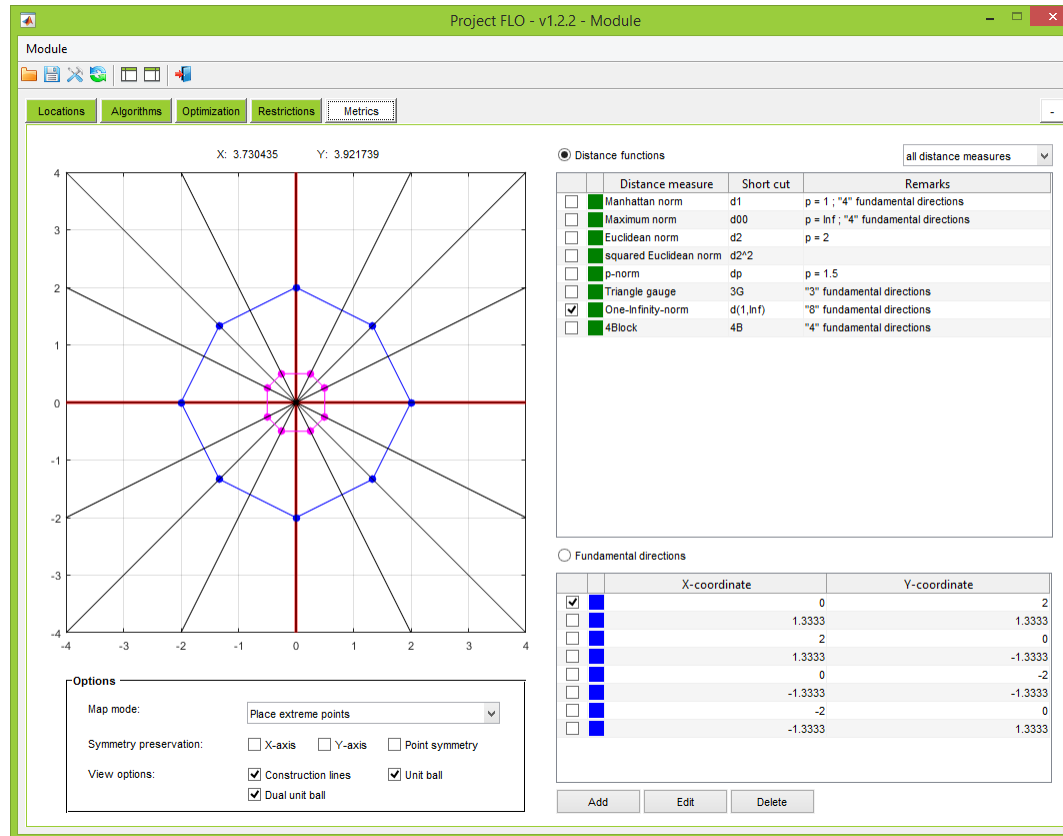
Module

Locations Algorithms Optimization Restrictions Metrics

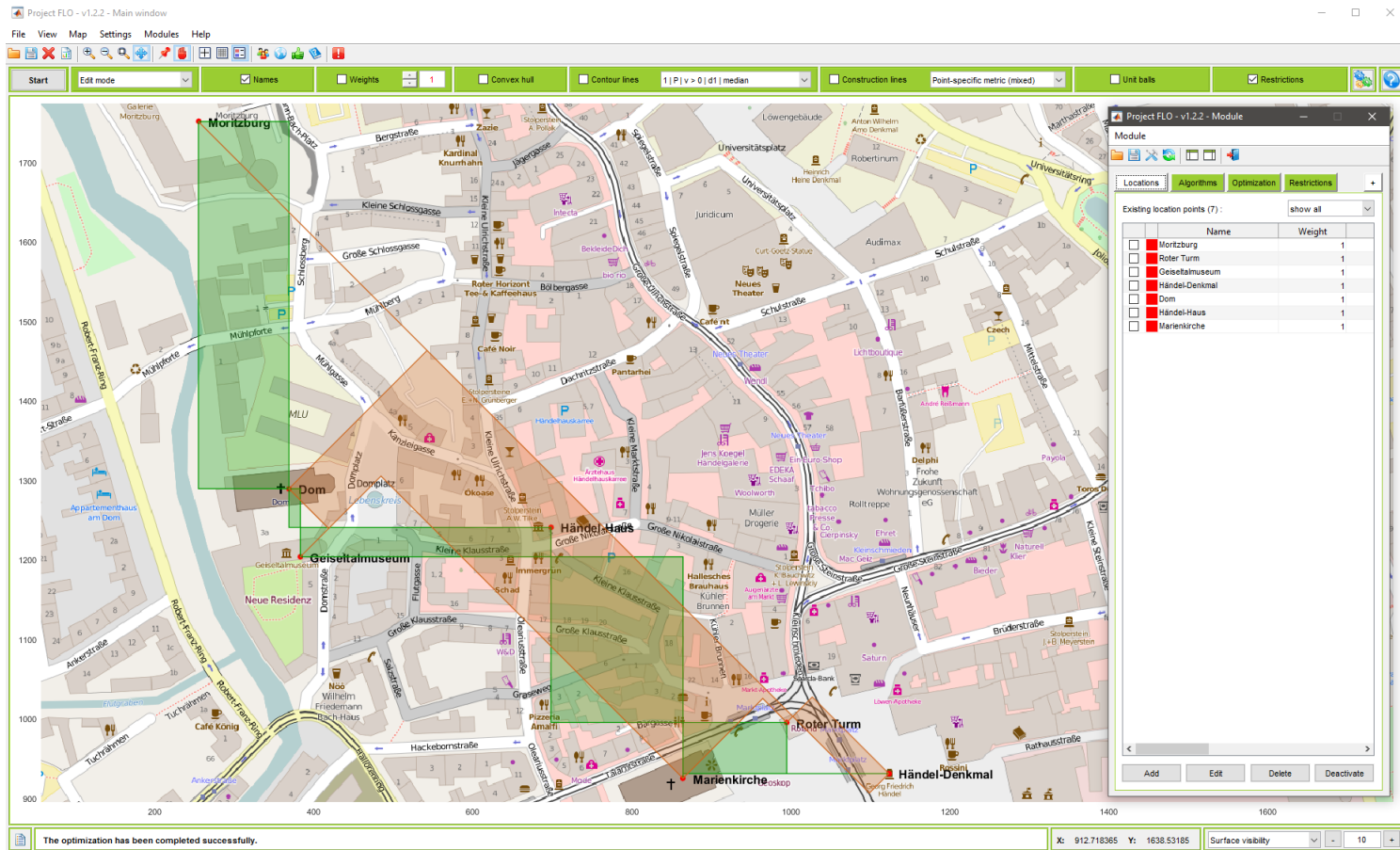
Total optimization time: 0.359375 s

Location problem	Runtime	Share	Solutions	Objective function value	
1 P v > 0 3G median	0.093750 s	2.14 %	X* = {(0.841345,-0.013392)}	f(x*) = 452.536790	Exact solution
1 P (+,-) (d00,d00) Eff-vector	0.078125 s	1.79 %	X* = {(1.53688979924675 0.68215260043704)} ...	f(x*) = -50.207953	Exact solution
1 P (+,-) (d1,d1) Eff-vector	0.062500 s	1.43 %	X* = {(1.53688979924675 0.68215260043704)} ...	f(x*) = -70.608071	Exact solution
1 P (+) d2^2 Eff-vector	0.046875 s	1.07 %	-	-	Exact solution
1 P (+) 4B Eff-vector	0.031250 s	0.71 %	-	-	Exact solution
1 P v > 0, X = R1 d00 median	0.031250 s	0.71 %	X* = {(0.807720258660029 0.510475175575593)}	f(x*) = 348.145169	Exact solution
1 P (+) d2 Eff-vector	0.031250 s	0.71 %	-	-	Exact solution
1 P (+) d00 wEff-vector	0.031250 s	0.71 %	-	-	Exact solution
1 P (+) d1 Eff-vector	0.031250 s	0.71 %	-	-	Exact solution
1 P v > 0, p = 1.5, eps = 1e-06 dp median	0.031250 s	0.71 %	X* = {(0.809227,0.680996)}	f(x*) = 425.205459	Approximative solution (nu)
1 P v > 0, X = R^2 \ Int(R1) d2^2 median	0.015625 s	0.36 %	X* = {(0.772438020562843 0.58669050437722)}	f(x*) = 306.613503	Exact solution
1 P v > 0, X = R^2 \ Int(R1) d1 median	0.015625 s	0.36 %	X* = {(0.798675,0.784366)}	f(x*) = 487.447617	Exact solution
1 P (+) d1 wEff-vector	0.015625 s	0.36 %	-	-	Exact solution
1 P v > 0, w < 0 d00 median	0.015625 s	0.36 %	X* = {(1.02681207238513 0.291383361850493)}	f(x*) = 313.237816	Exact solution
1 P v = 1 d2 center	0.015625 s	0.36 %	X* = {(0.571531,0.557062)}	f(x*) = 0.977764	Exact solution
1 P v > 0 d2 median	0.015625 s	0.36 %	X* = {(0.833026,0.586009)}	f(x*) = 398.644010	Approximative solution (nu)
1 P v > 0 d00 median	0.015625 s	0.36 %	X* = {(0.807720,0.510475)}	f(x*) = 348.145169	Exact solution
1 P (+) d00 Eff-vector	0.000000 s	0.00 %	-	-	Exact solution
1 P v > 0, w < 0 d1 median	0.000000 s	0.00 %	X* = {(0.798674522217184 0.784366450381243)}	f(x*) = 442.166012	Exact solution
1 P v > 0 d00 center	0.000000 s	0.00 %	X* = conv{(0.539588175459584 0.238169827426...}	f(x*) = 78.786828	Exact solution
1 P v > 0 d1 center	0.000000 s	0.00 %	X* = conv{(0.308386189005701 0.406002693842...}	f(x*) = 118.867470	Exact solution
1 P v > 0 d2^2 median	0.000000 s	0.00 %	X* = {(0.772438,0.586691)}	f(x*) = 306.613503	Exact solution
1 P v > 0 d1 median	0.000000 s	0.00 %	X* = {(0.798675,0.784366)}	f(x*) = 487.447617	Exact solution

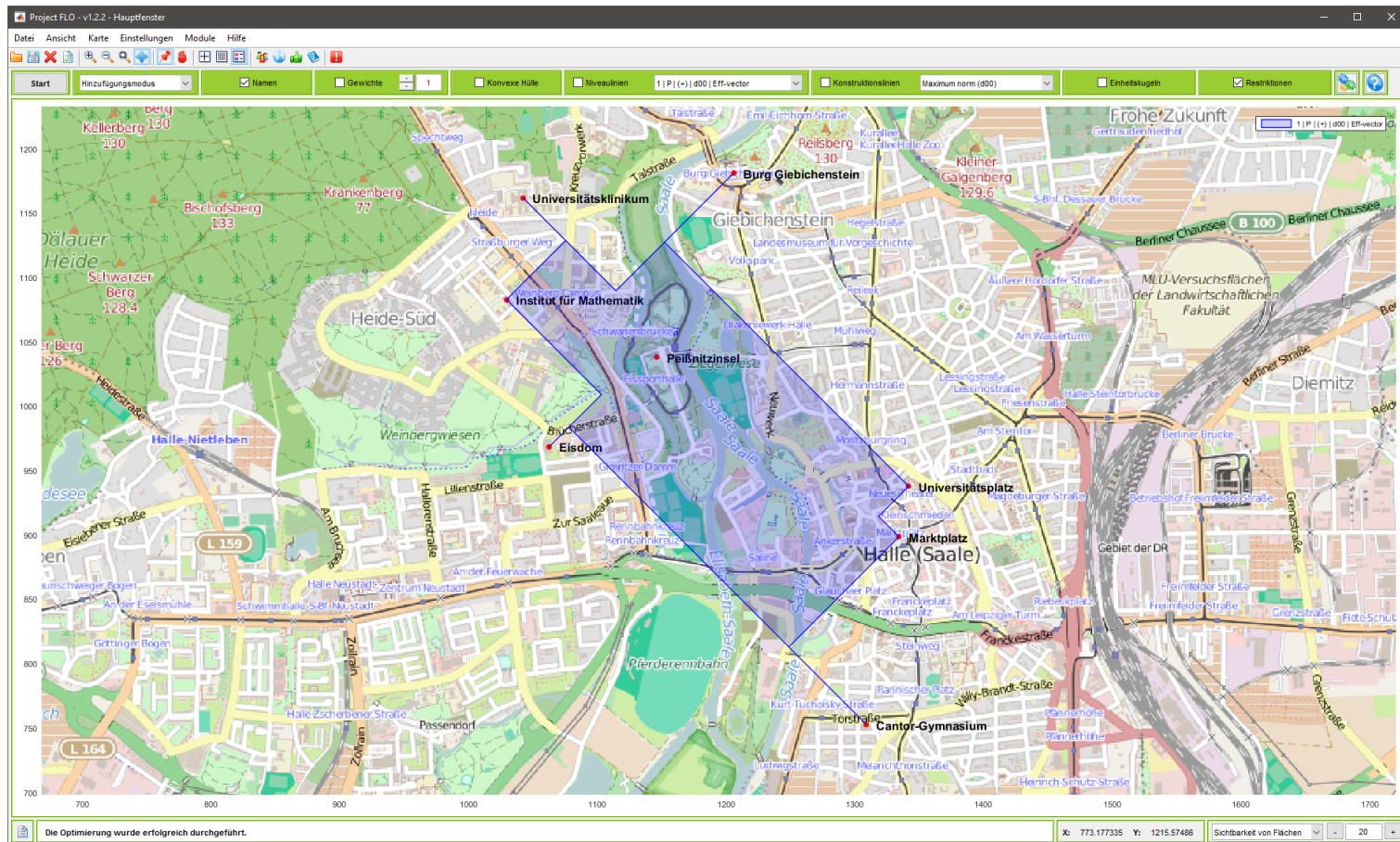
Software FLO: Optimization. Determine the solutions of the location problem using different algorithms depending from the type of the problem.



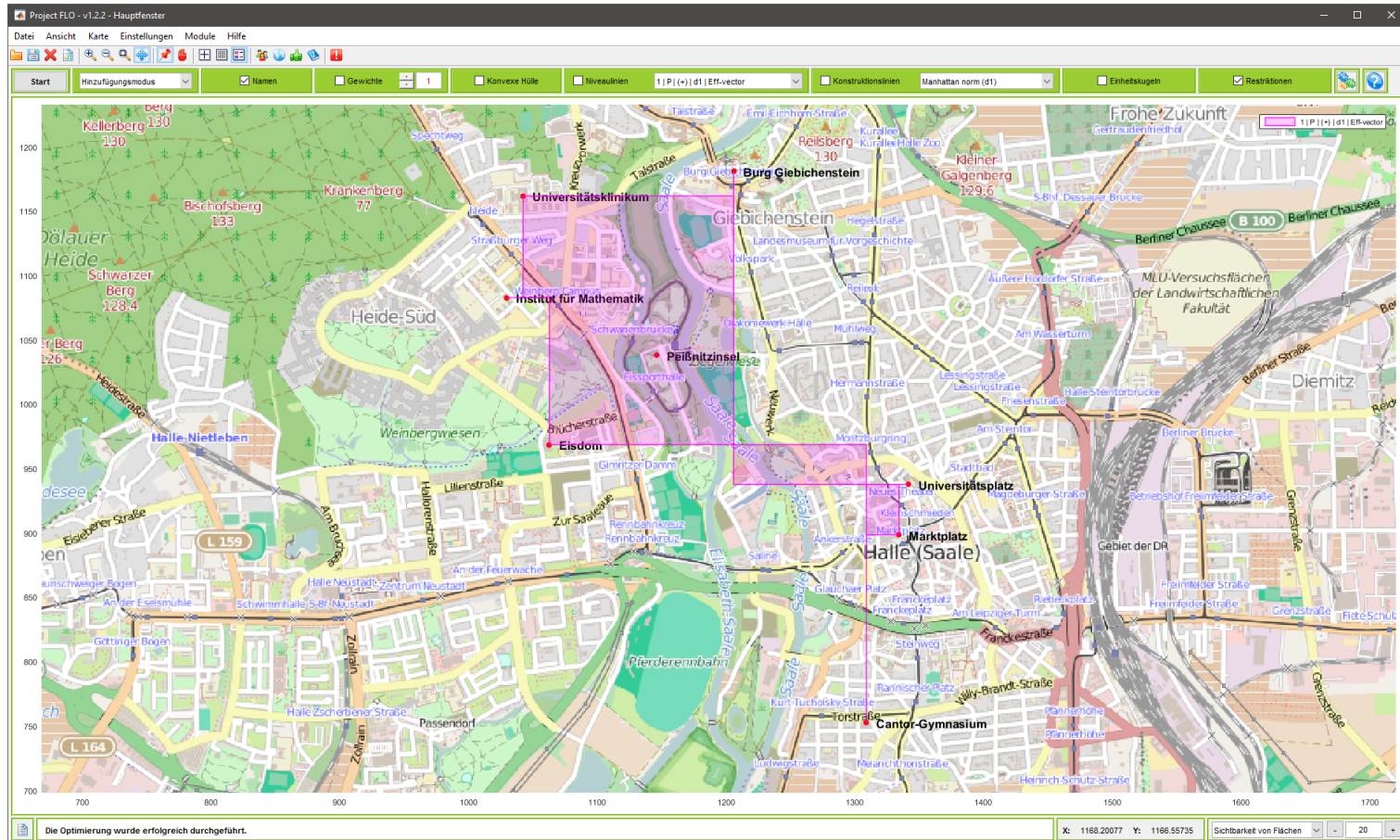
Software FLO: Metrics. Extreme points, level sets, unit balls.



Software FLO: Set of efficient elements $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$ of (POLP) with the maximum norm (red color) and with the Manhattan norm (green color) and locations.



Software FLO: The set of efficient elements $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the maximum norm.



Software FLO: The set of efficient elements $\text{Eff}_{\text{Min}}(\mathbb{R}^2 \mid f)$ of the multiobjective location problem (POLP) with the Manhattan norm.

3. Conclusions

Enhancements to the software FLO:

- To include algorithms for solving **constrained** multiobjective location problems.
- To include algorithms for solving **multiobjective approximation problems**.
- To derive **new and efficient algorithms** for solving multiobjective location and approximation problems.

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