

Proof of Galerkin orthogonality

$$\begin{aligned} u \text{ solves} & \quad a(u, v) = F(v) \quad \forall v \in V_0 \\ u_S \text{ solves} & \quad a(u_S, v) = F(v) \quad \forall v \in S \end{aligned}$$

$$a(u - u_S, v) = 0 \quad \forall v \in S$$

Quasi-Optimality:

$$\begin{aligned} c \|u - u_S\|_V^2 & \stackrel{\text{coercivity}}{\leq} a(u - u_S, u - u_S) \\ & = a(u - u_S, u) - \underbrace{a(u - u_S, u_S)}_0 \\ & \stackrel{\text{Gal-orth.}}{=} a(u - u_S, u) - a(u - u_S, v) \\ & = a(u - u_S, u - v) \\ & \leq C \|u - u_S\|_V \|u - v\|_V \end{aligned}$$

$$\Rightarrow \|u - u_S\|_V \leq \frac{C}{c} \inf_{v \in S} \|u - v\|_V \quad \square$$

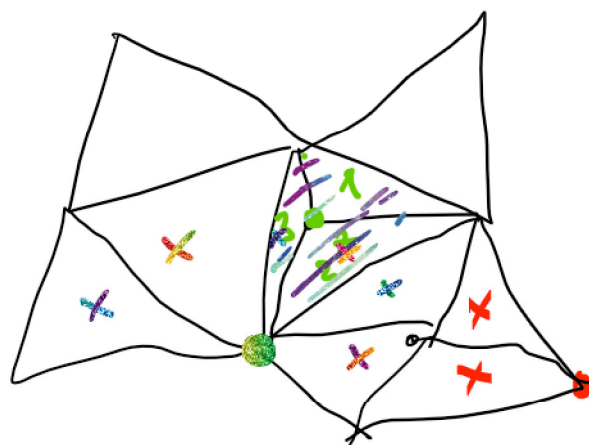
Convergence rates under the assumption that u is **regular**; $u \in H_0^1(\Omega) \cap H^{p+1}(\Omega)$

$$\inf_{v \in S} \|u - v\|_V \leq \|u - \text{Int}_S^p(u)\|_V$$

$$\inf_{u \in S} \|u - u_h\|_V = \inf_{u \in S} \|u - u_h\|_V \leq C_{int} h^p \|u\|_{H^{p+1}(\Omega)}$$

→ Convergence with rates:

$$\|u - u_h\|_V \leq \frac{C}{c} C_{int} h^p \|u\|_{H^{p+1}(\Omega)}$$



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