MAT563 Category Theory and its Applications

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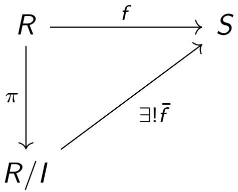
Introduction

- You've probably noticed this pattern in pure maths: we often consider sets with some structure (**vectors spaces, groups, algebras, topological manifolds**...) and maps between them (**isomorphisms, homomorphisms, homeomorphisms**...) - this is the starting point for **category theory**!
- The idea is to put these into a common framework, as objects and morphisms of a **category** (**of vector spaces, of abelian groups, of topological spaces**...)
- **Why care about this abstraction?** It makes it easier to see how superficially unrelated objects (e.g. geometric vs algebraic) closely relate. It is actually quite intuitive and in recent years proved useful not only in **pure maths**, but also **physics, computer science and beyond**.
- **This is going to be an introductory seminar - you need only basic background in algebra** (Algebra I), **knowledge of topology and geometry is useful but not essential**

Category theory taster

- A **category** C is a set of **objects** $\mathrm{Ob}\, C$ and **morphisms (arrows)** $\mathrm{Mor}\, C$ between them, that can be composed.
- The language is that of **commutative diagrams**, which show **how the objects are related by the arrows**
- **Example: universal property of the quotient ring (in the category of commutative rings)** let R, S be commutative rings, $I \leq R$ an ideal of R, π the quotient map, f any ring homomorphism such that $f(I) = \{0\}$. Then



You can follow arrows from R, S in any order and the result is the same i.e. for each homomorphism f, there exists the unique homomorphism $\bar{f}$.

Overview and requirements

The topics we roughly intend to cover are:

- Basic notions: categories, functors, natural transformations
- Constructions in categories: limits, colimits, universals
- Algebraic properties: monads, algebras, monoids
- Applications: geometry and topology, computer science and ML, physics

The main textbook will be the category theory bible *Categories for the Working Mathematician*, by S. MacLane, supplemented with other sources, including on LaTeX for diagrams.

To pass the seminar, everyone needs to give one talk, and produce corresponding chapter of LaTeX notes.

More details coming soon! If you're interested or have any questions, feel free reach me at
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