# Course Introduction

## Mathematics Electives in fall semester 2024

# Overview

- MAT 979 Seminar in Combinatorics and Geometry – Tumarkin
- MAT 817 Seminar Semiclassical Spectral Estimates – Oldenburg
- MAT 602 Functional Analysis – Schlein (Oldenburg)
- MAT 565 Representation Theory – Beliakova (Andreev)
- MAT 633 Field Theory for Mathematicians – Cattaneo
- MAT 922 Probability II – Rosales Ortiz
- MAT 933 Complex networks: theory and applications – Bovet
- MAT 959 Seminar in Data Science and Mathematical Modeling – Bovet
- *STA 222 (M3L101) Computers and Computing – Furrer*
- *STA 121 Statistical modeling*
- *STA 472 Good statistical practice*
- MAT 820 Numerisches Praktikum – Sauter
- MAT 828 Numerical Methods of ODEs – Sauter
- MAT 784 Seminar on Knot Theory – Wildi

# MAT 979
# Seminar in Combinatorics and Geometry – Tumarkin

# Combinatorics and geometry

The seminar will cover a selection of topics in the intersection of combinatorics and geometry:

- Counting geometric objects
- Geometric problems with a ‘combinatorial feel’

What will you learn?

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Examples of topics:

# Catalan numbers

**Question:** in how many ways can one triangulate a regular $n$-gon by non-intersecting diagonals?

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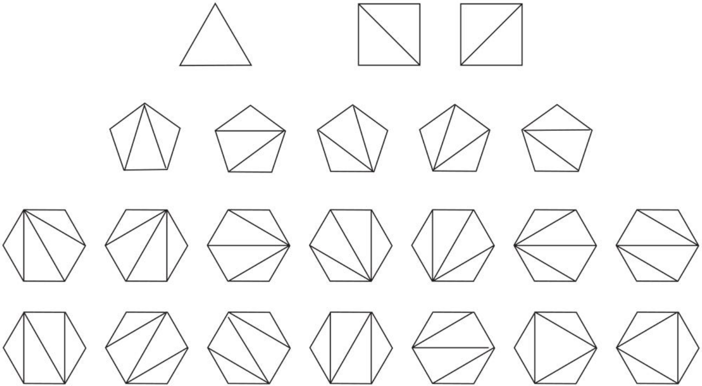

# Wallpaper groups

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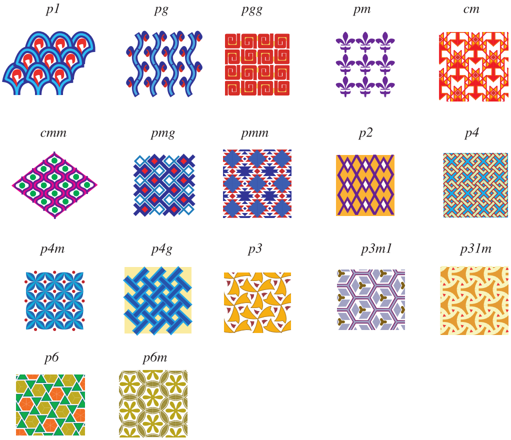


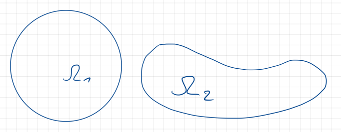
# MAT 817 Seminar
# Semiclassical Spectral Estimates – Oldenburg

# Schrödinger Operator

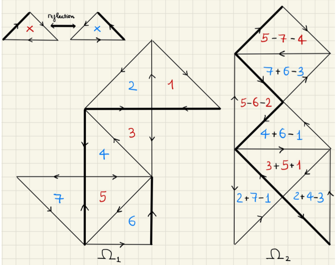
- $[-\Delta + V(x)]f(x) = \lambda f(x)$ on $L^2(\mathbb{R}^d)$, $V \in L^p(\mathbb{R}^d)$
- Question: How many negative eigenvalues are there?
- Motivation: Quantum Mechanics
- Answer: At most $C_d \int |V_-|^{d/2} dx$, $d \geq 3$
- More general: Lieb-Thirring inequalities

# Laplace Operator on a Bounded Domain

- $-\Delta f = \lambda f$ on $L^2(\Omega)$, $\Omega \subset \mathbb{R}^d$
- $d = 2$: eigenvalues are frequencies of a drum
- Question: Can one hear the shape of a drum?




- Answer: No



# Formalities

- Prerequisites: Functional Analysis
- Material: Lecture Notes from Prof. Phan Thành Nam (Chapter 1-6)
  math.lmu.de/∼nam/LectureNotesFA2021.pdf
- Additional: 'Schrödinger Operators: Eigenvalues and Lieb–Thirring Inequalities' by Rupert L. Frank, Ari Laptev, Timo Weidl

# MAT 602 Functional Analysis – Schlein

# MAT602 Functional Analysis

## Basic information

**Teacher**: Benjamin Schlein

**Time**: Mo 13-15, Th 10-12

**Exercises**: 2 hours per week, time will be fixed later on

**Prerequisites:** Analysis 1,2,3, Linear Algebra 1,2.

**Language**: English

**Exam**: oral or written, depending on number of students

## What is functional analysis?

Functional analysis is linear algebra on infinite dimensional vector spaces (often spaces of functions).

In contrast to linear algebra, analytic ideas (convergence of sequences, continuity of functions, ..) play central role.

## Why should you follow this class?

Modern analysis focus on partial differential equations posed on infinite dimensional function spaces. Functional analysis is absolutely crucial to study these problems.

Functional analysis plays a very important role in modern probability theory, financial mathematics, ..

# Content

**Spaces and structures**: Banach and Hilbert spaces, with several examples of function spaces.

**Compactness**: when are sets in infinite dimensional function spaces compact? Arzela-Ascoli and Riesz Theorems.

**Classical functional analysis**: Hahn-Banach Theorem, Baire Category Theorem and their consequences.

**Weak Topologies:** weak convergence, weak-* topology and compactness (Banach-Alaoglu Theorem).

**Spectral theory**: spectral properties of compact operators, spectral theorem for bounded self-adjoint operators.

## Literature

There will be a script.

Other books that may be useful include:

- H.W. Alt. Lineare Funktionalanalysis. Springer. (German)

- M. Reed, B. Simon. Funtional analysis. Mathematical Methods in Physics, Vol. I. Elsevier. (English)

- E. Lieb, M. Loss. Analysis. Graduate Studies in Mathematics (AMS). (English)

- B. Bollobas. Linear Analysis. Cambridge Mathematical Textbooks. Cambridge University Press. (English)

# MAT 565 Representation Theory - Beliakova

Prof. Dr. A. Beliakova Herbstsemester 2024

# Introduction to Representation Theory

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In Linear Algebra we study $\mathbf{k}$-vector spaces and linear maps between them. They form a category $\mathrm{Vect}_{\mathbf{k}}$ where $\mathbf{k}$ is a field. In practice, many models have certain symmetry, mathematically described by an action of a group or algebra $A$ on vector spaces. Then we would like to understand maps that preserve this action. Hence, the main object of study will be the category of representation of an algebra $A$, usually denoted $\mathrm{Rep}A$.

In details, a vector space $V$ equipped with a map

$$\rho : A \to \operatorname{End} V$$

which is a homomorphism of algebras is called a *representation* of $A$ or an $A$*-module*. A representation is called *irreducible* if it has no proper non-trivial submodules.

Like Linear Algebra culminates in Jordan decomposition theorem, the representation theory of finite-dimensional algebras has two important structural results – the Jordan–Hölder theorem and the Krull–Schmidt theorem. We will prove both of them. In addition, we will characterise any finite-dimensional algebra in terms of the endomorphism rings of its representations:

$$A \setminus \mathrm{Rad}(A) = \bigoplus_i \operatorname{End} V_i$$

where $V_i$ are irreducible representations and $\mathrm{Rad}(A)$ is the largest nilpotent two-sided ideal of $A$.

Next we develop the classical representation theory of finite groups. We prove Burnside's theorem and Frobenius's divisibility and character formulas. Finally, we introduce a new powerful tool – the Schur–Weyl duality, and use it to identify representation theories of the symmetric group and $\mathrm{GL}(V)$.

The course will follow Etingof's Lecture Notes (https://math.mit.edu/ etingof/repb.pdf, Chapters 1-5) and is suitable for bachelor, master and PhD students completed Linear Algebra I,II and Algebra successfully.

# MAT 633
# Field Theory for Mathematicians - Cattaneo

# Field Theory for Mathematicians

Wednesdays, 13:00 - 17:00

(Exercises: Mondays, 15:00 - 17:00)

advanced BSc, MSc, PhD

Notes will be provided during the course

# What is it about?

Physics is mathematically modeled by "fields"

and their evolution

Example: The electromagnetic field – a pair of vector fields

$$\vec{E}, \vec{B}$$

+ Maxwell's equations

I will focus on the action functional approach:

$S$ is a function on the space of fields

- classical FT: $\delta S = 0$ (critical points)
  - ↳ PDEs governing the evolution
- quantum FT: " $e^{\frac{i}{\hbar} S} d\mu$ " as a "probability amplitude measure" on the space of fields

I will introduce you to classical and quantum field theory

(with a short overview of classical and quantum mechanics)

focusing on the topological, geometrical, and algebraic aspects

(less on the analytical aspects)

- No prerequisites in physics
- Motivation for
  - differential geometry
  - super geometry
  - homotopy theory
  - ...

# MAT 922 Probability II – Rosales Ortiz

I - Conditional Expectation

II - Martingales

III - Markov Chains

Prerequisites: Analysis III (Measure theory) and Probability I.

Bibliography: Le Gall, J.-F. *Measure Theory, Probability, and Stochastic Processes.*

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# I - Conditional Expectation

• Informally, conditional expectation gives a rigorous framework to the rather intuitive idea of "the expected value of $X$ given that we have observed $Y$"

$$\mathbb{E}(X|Y)$$

In contrast with $\mathbb{E}(X)$, the conditional expectation $\mathbb{E}(X|Y)$ is (a priori) itself a non-trivial random variable! (since it is a function of $Y$).

Prerequisites: Analysis III (Measure theory) and Probability I.

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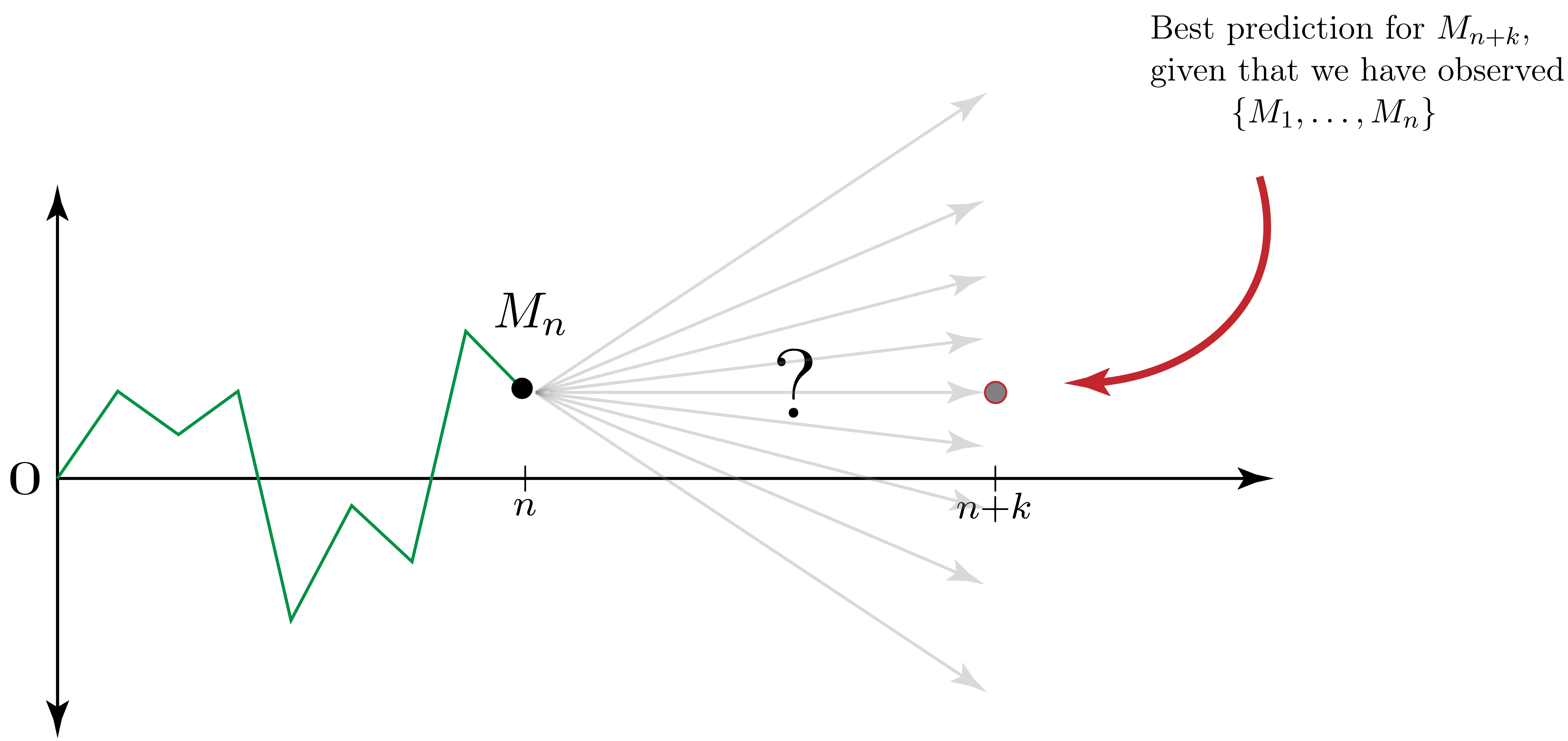
# II - Martingales

Discrete time process $M = (M_n)_{n\geq 1}$ which, informally, satisfies that given that we have observed the positions $M_1, \ldots, M_n$ of $M$, our best prediction for its position at time $n+k$ is that it will be at the last observed position $M_n$. Formally,

$$\mathbb{E}(M_{n+k}|M_1, \ldots, M_n) = M_n.$$



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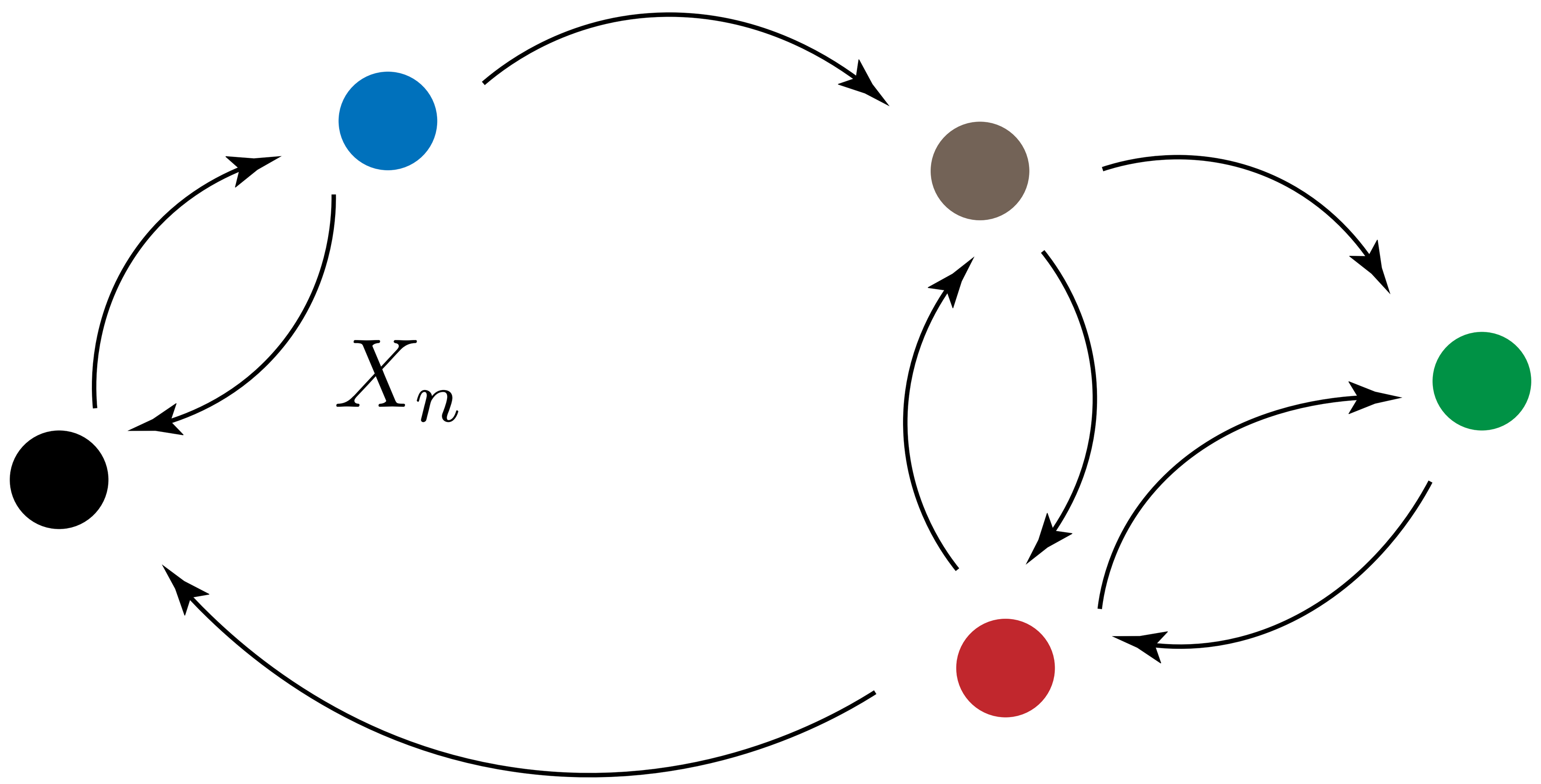
Processes indexed by $\mathbb{N}$ satisfying that the evolution of the future of the process, given the past, only depends on the present state. This is known as the Markov property.



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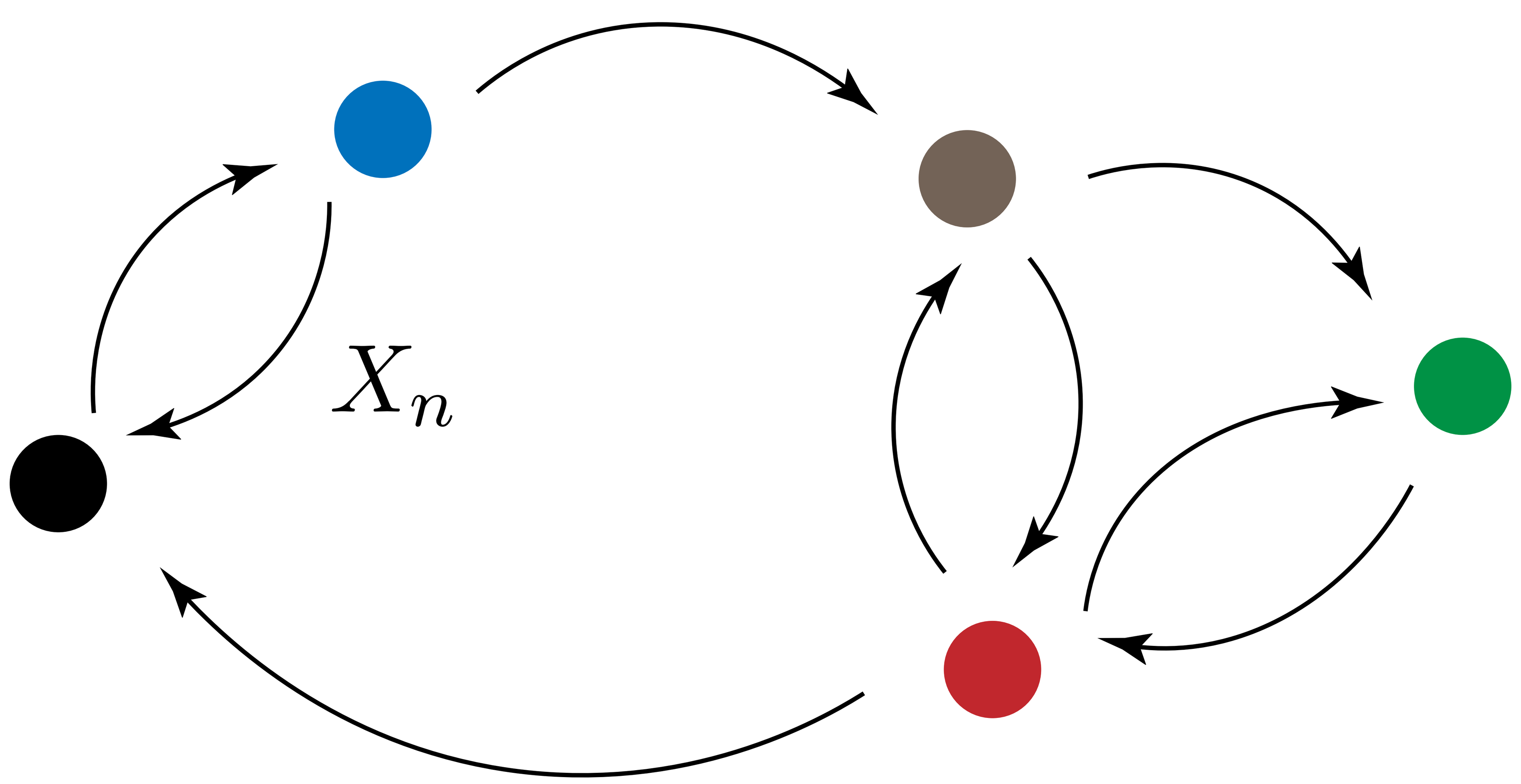


(i) Time $= \mathbb{N}$
(ii) State space $=$ some discrete subset
(iii) The Markov property:

$$\mathbb{P}_{\bullet}(X_{n+1} \in \cdot \mid X_1 = \bullet, \ldots, X_n = \bullet)$$

$$= \mathbb{P}_{\bullet}(X_1 \in \cdot)$$

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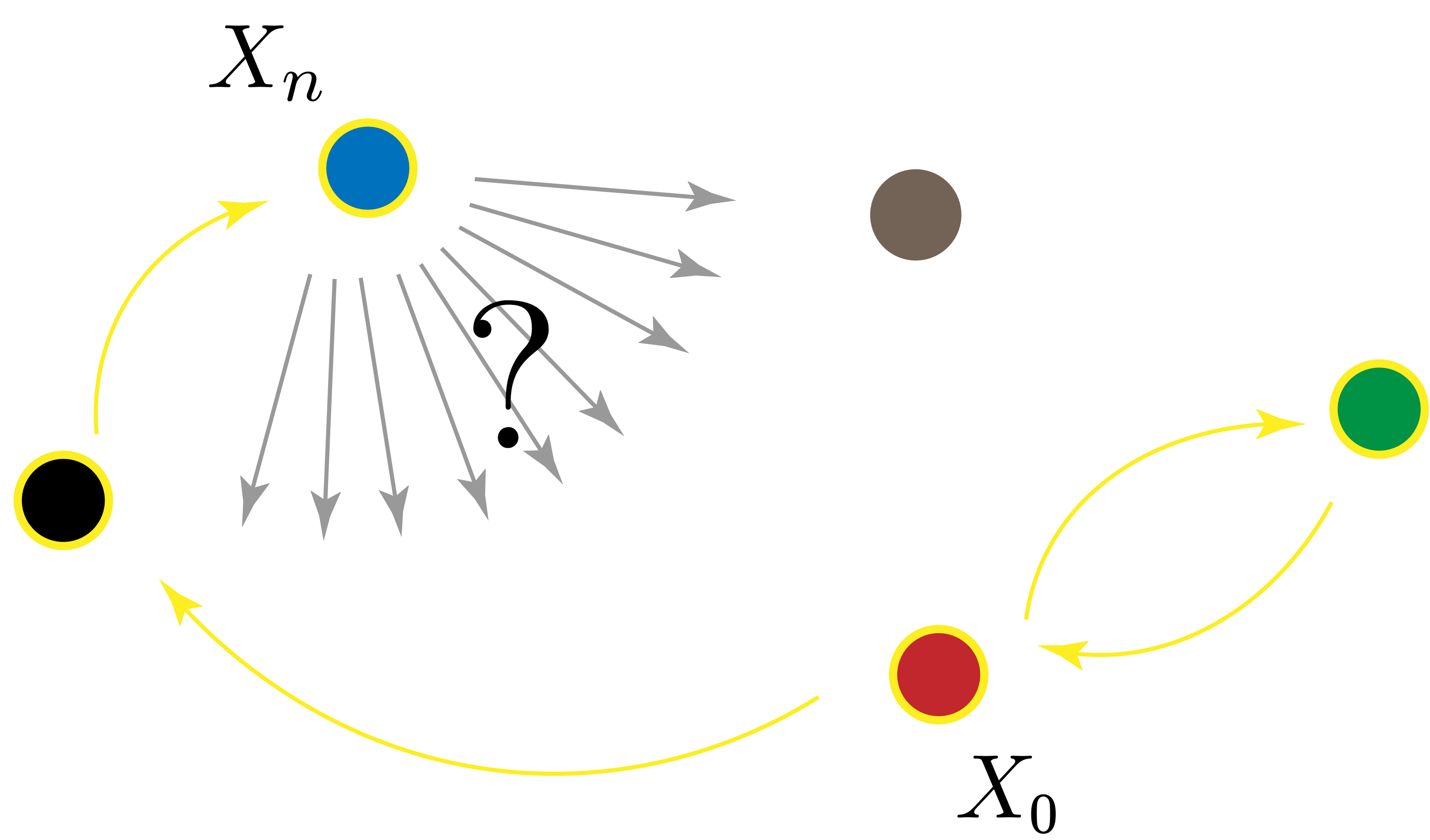


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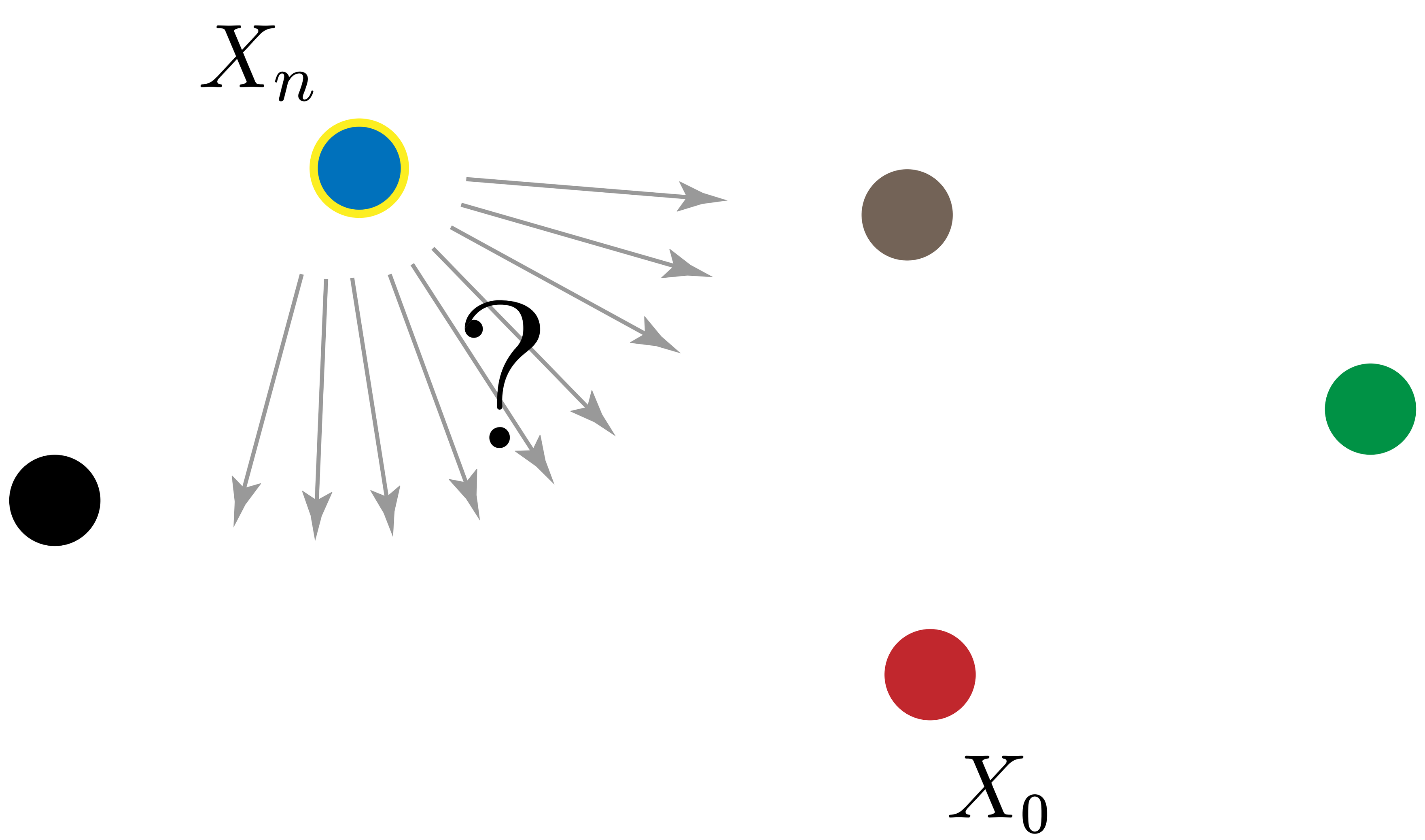


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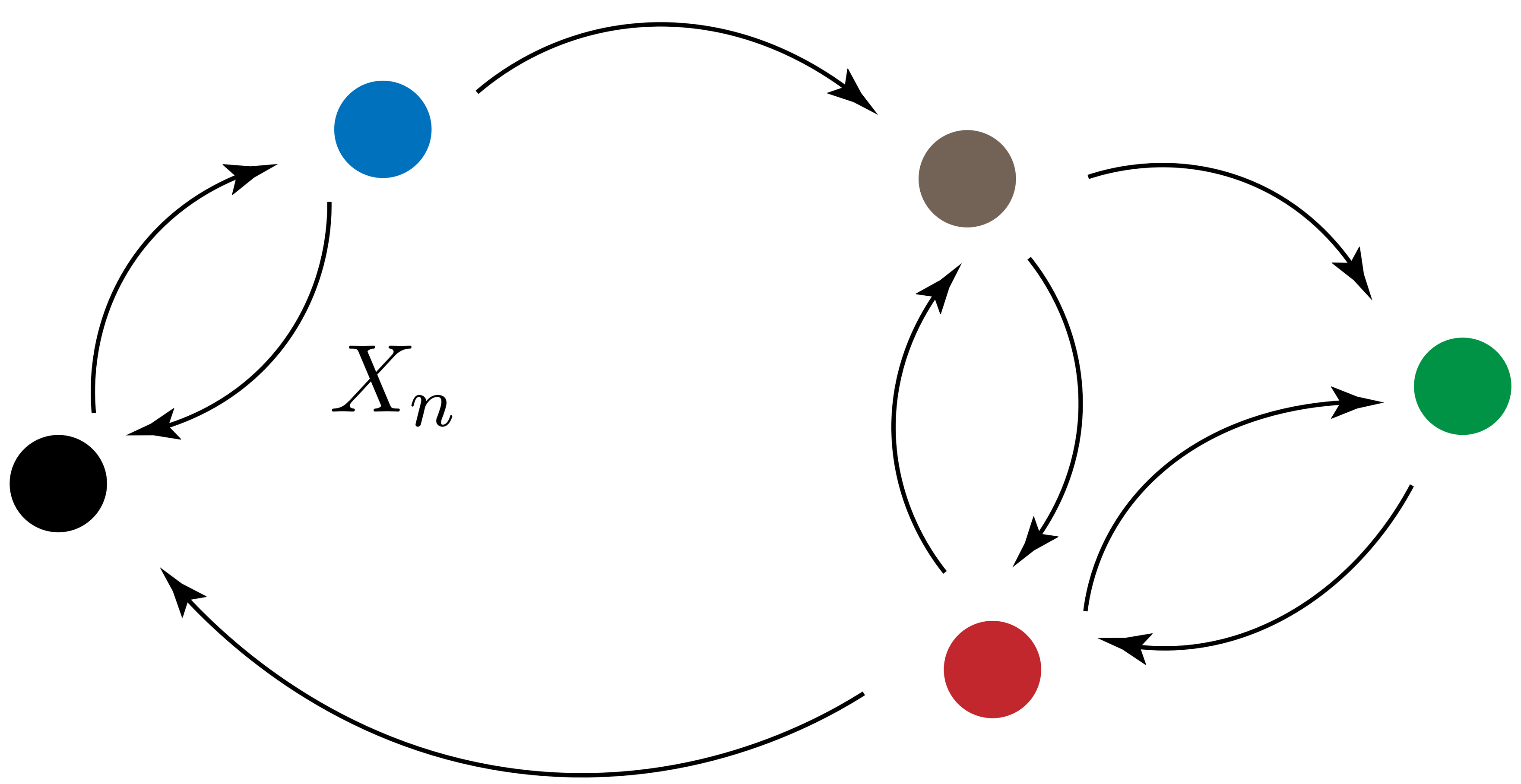


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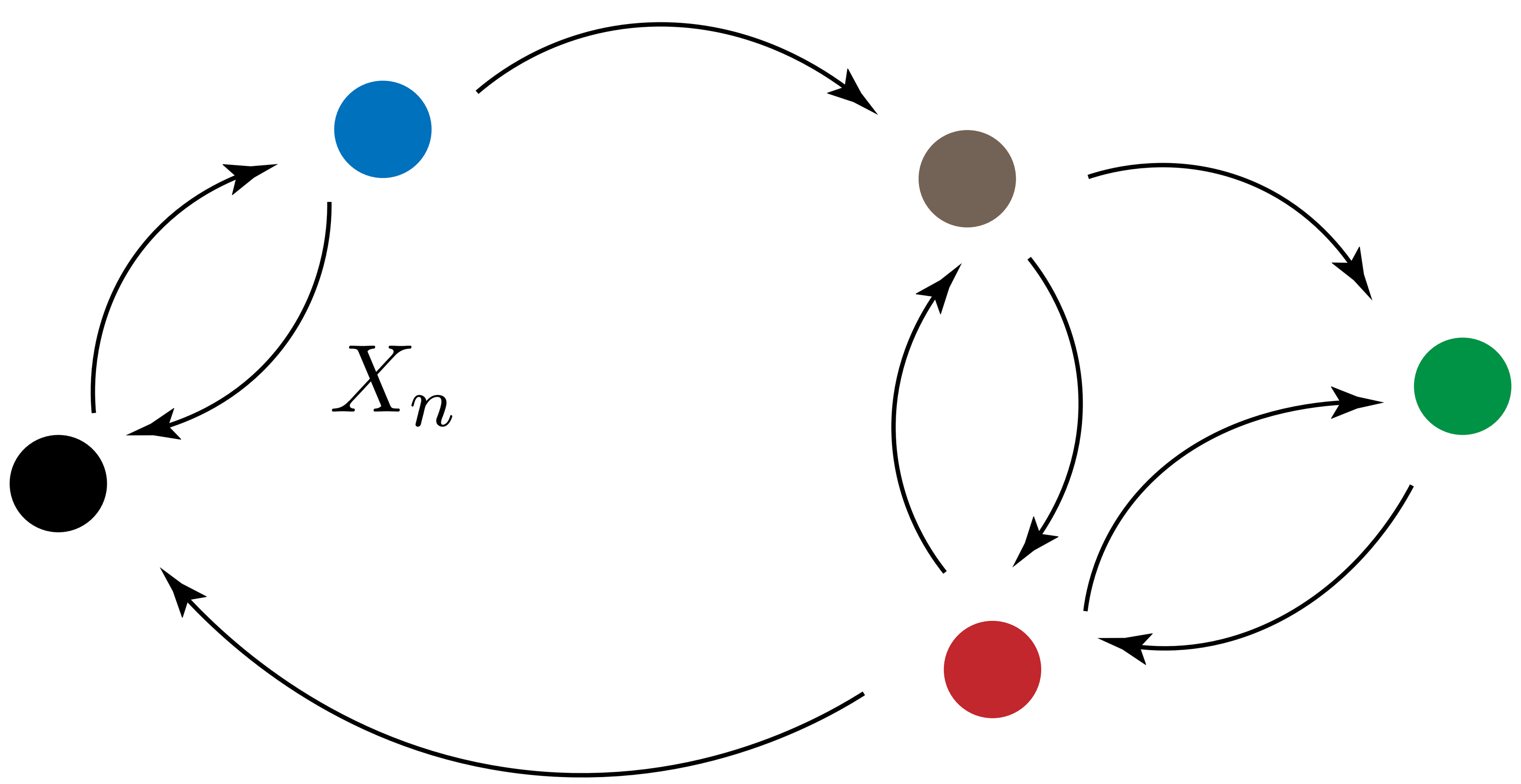


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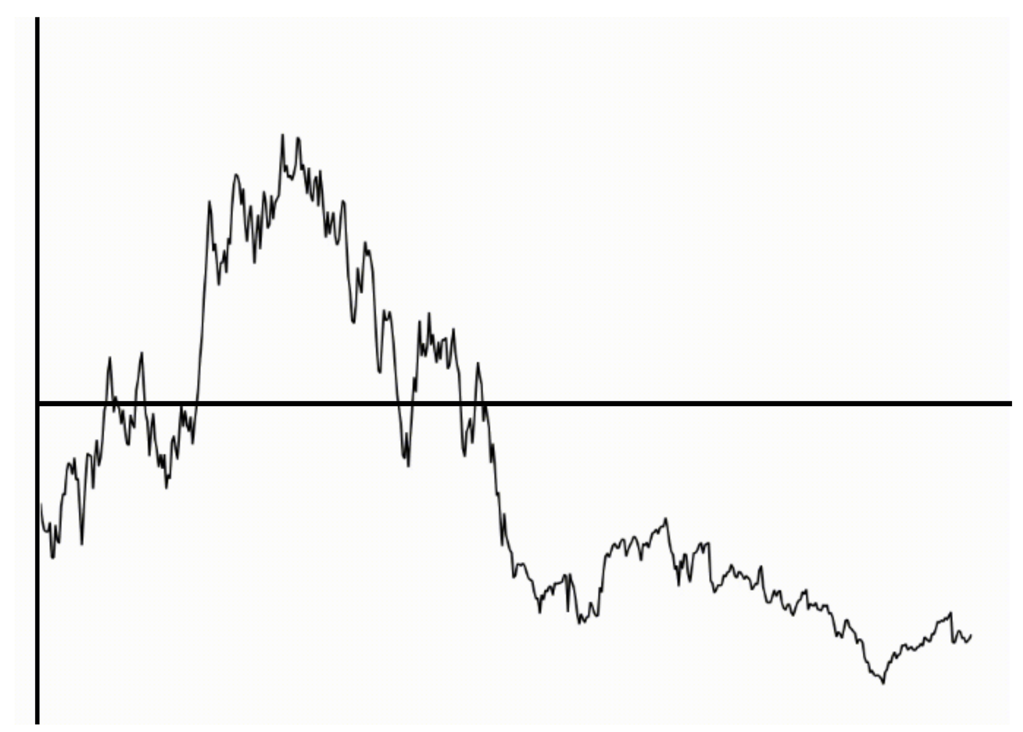

Brownian Motion

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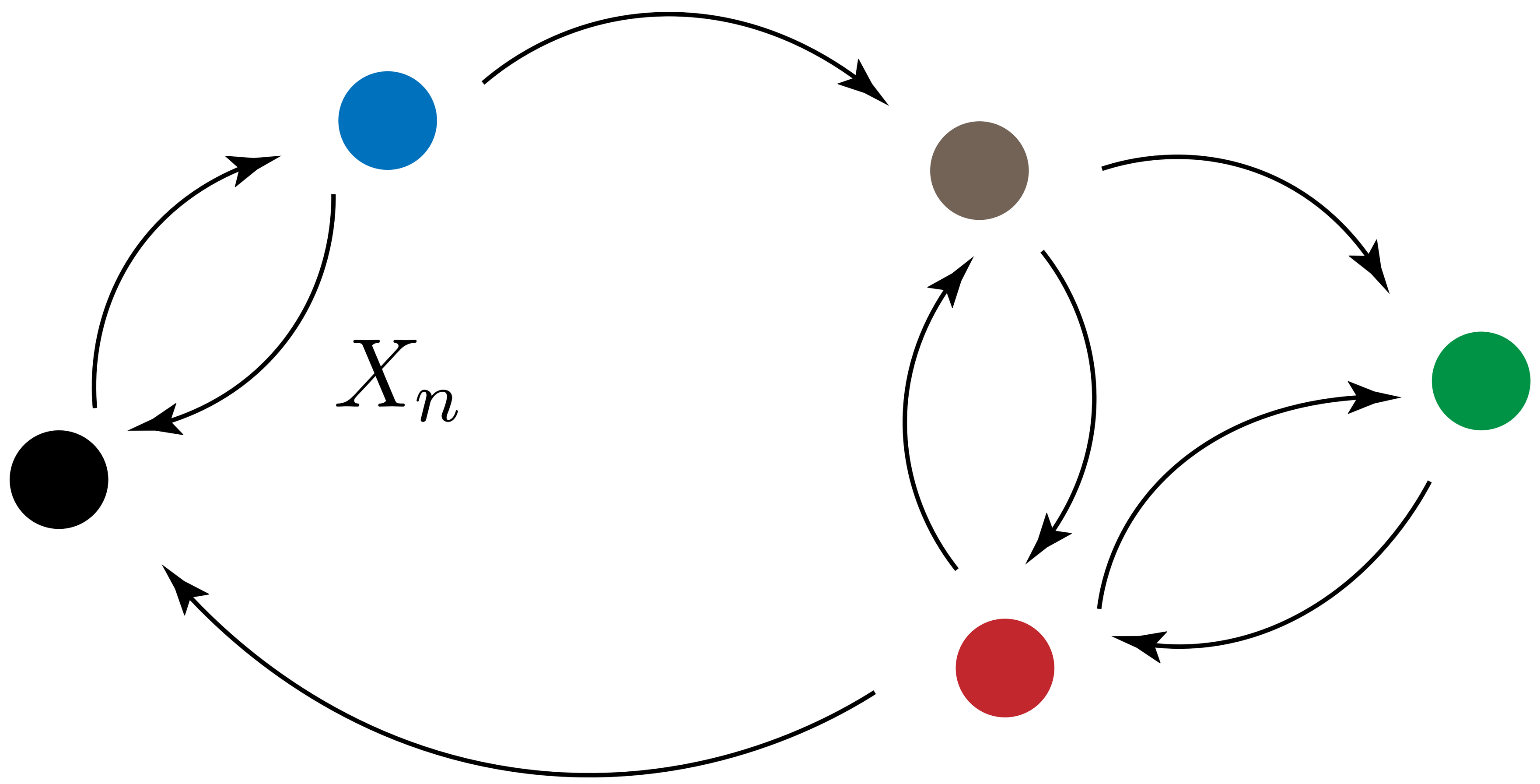


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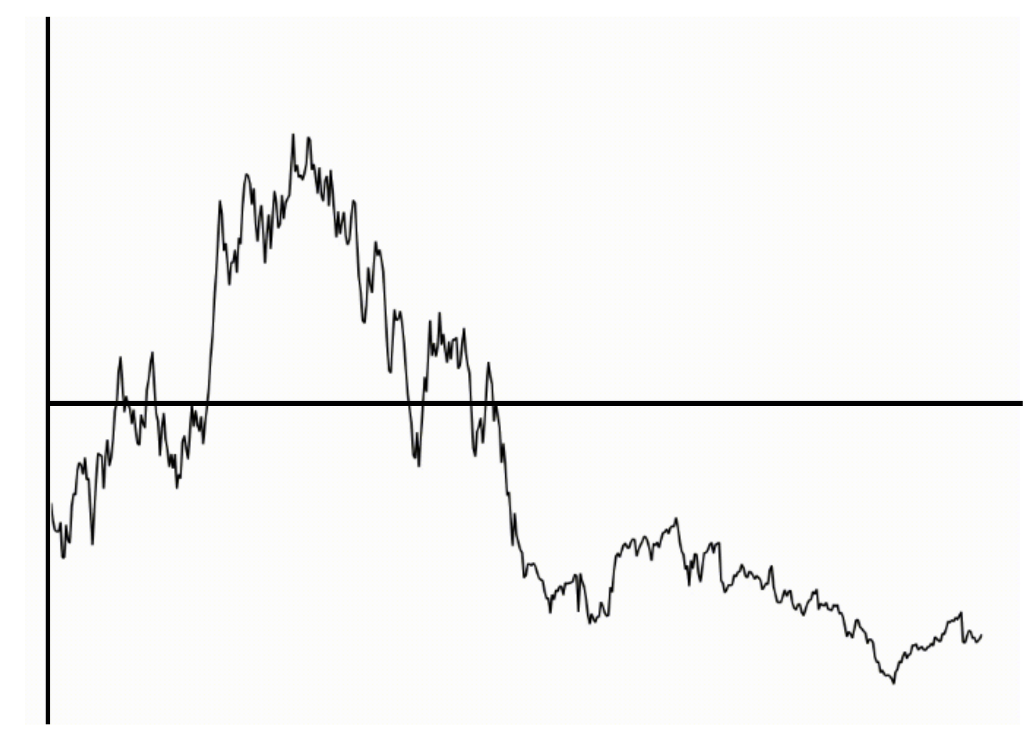
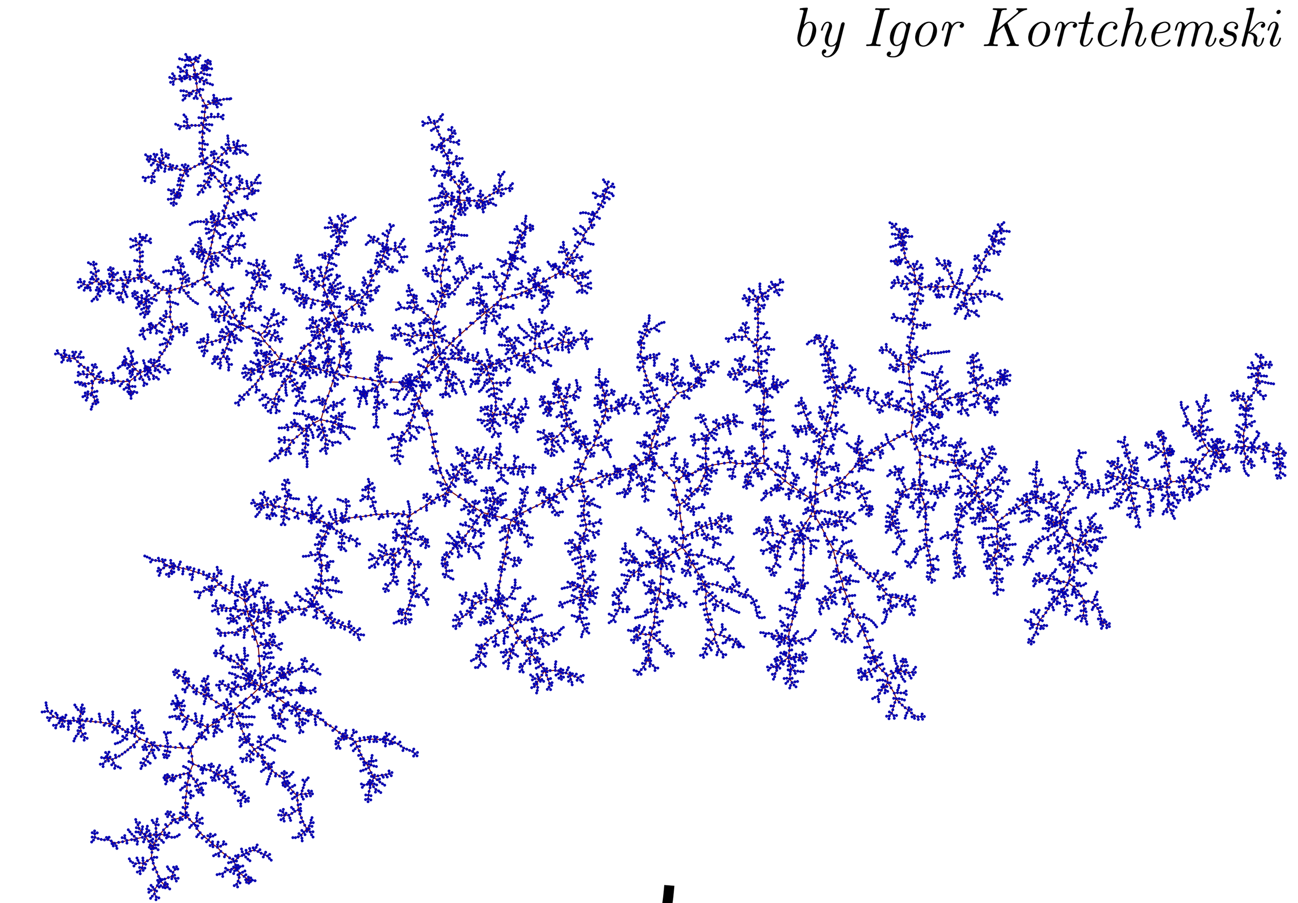
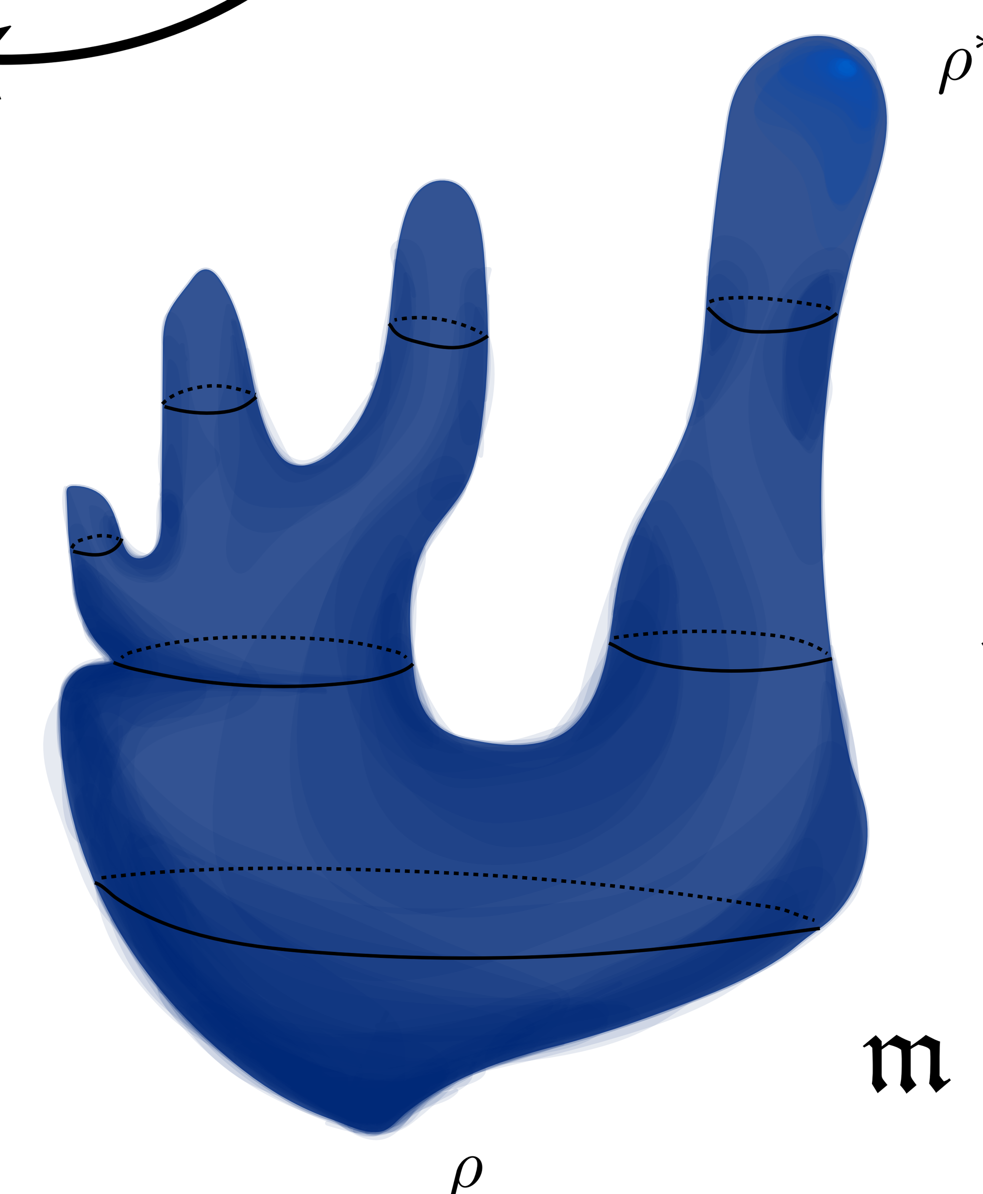


Brownian Motion

Brownian tree
*by Igor Kortchemski*

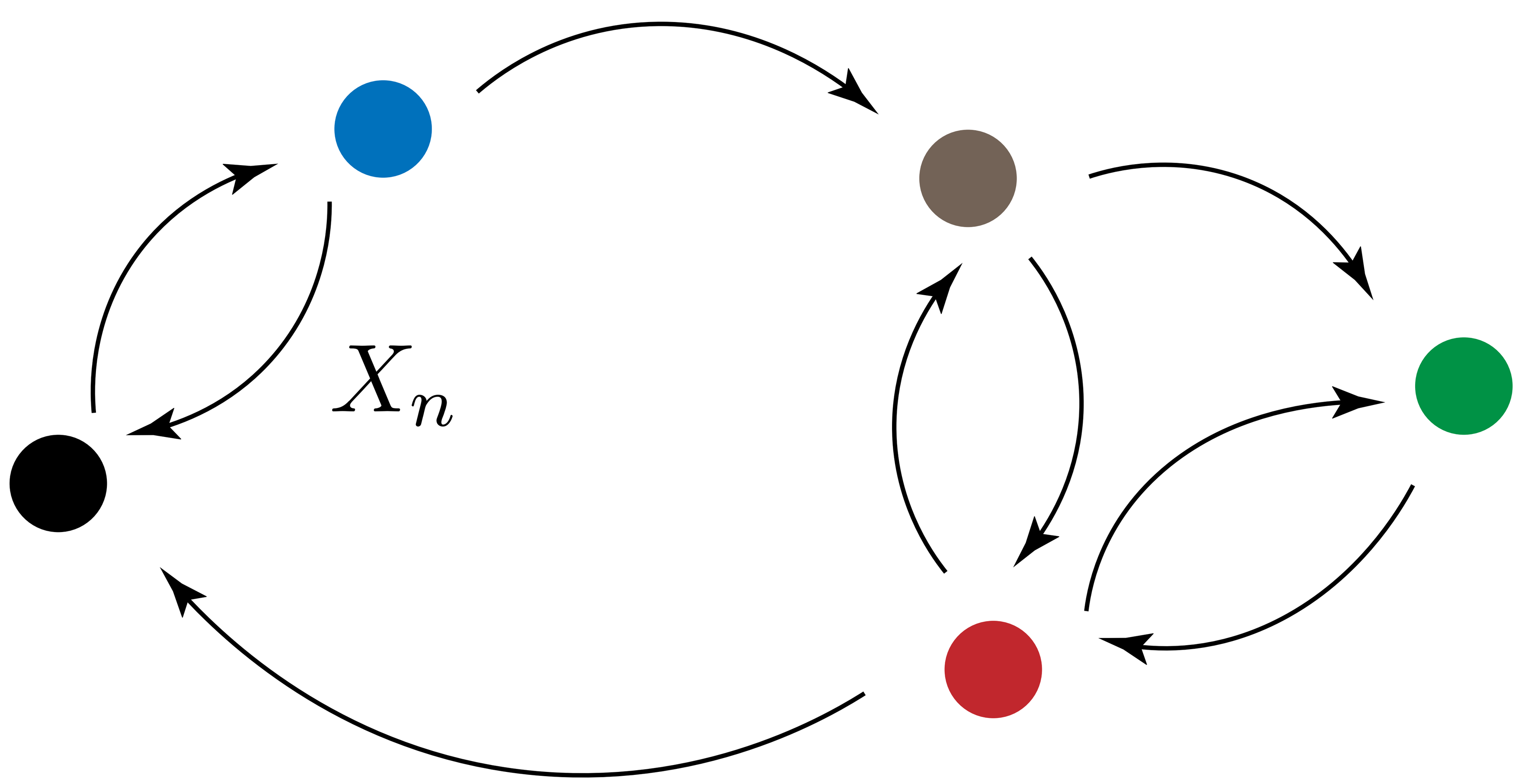



Excursion of Brownian motion indexed by the Brownian tree

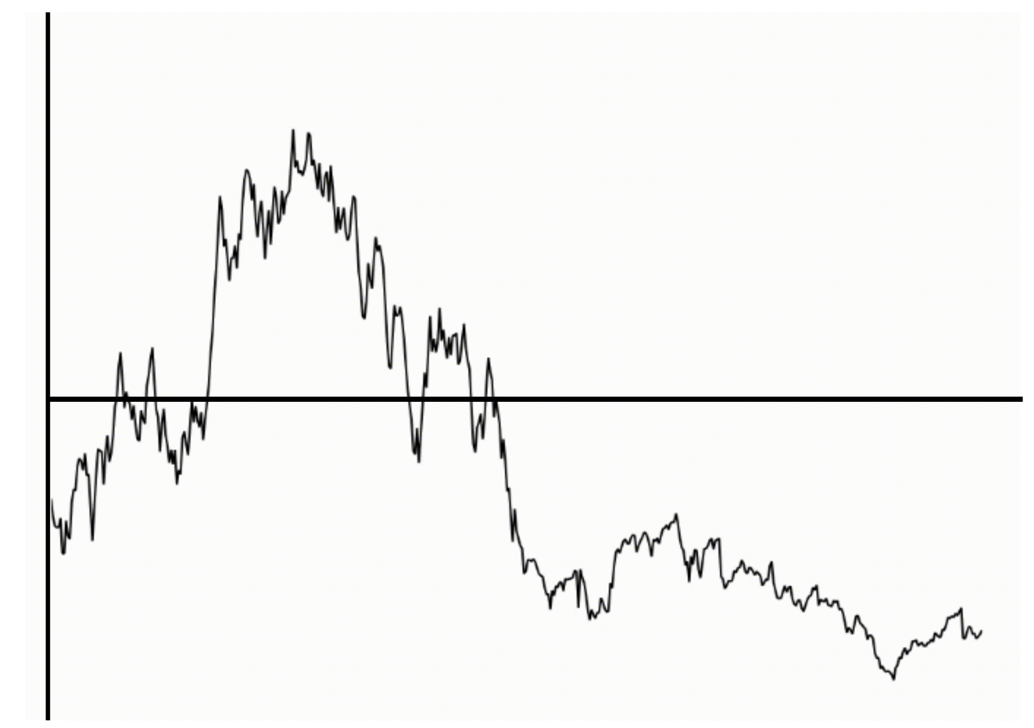



Brownian sphere

# III - Markov Chains




Markov chains



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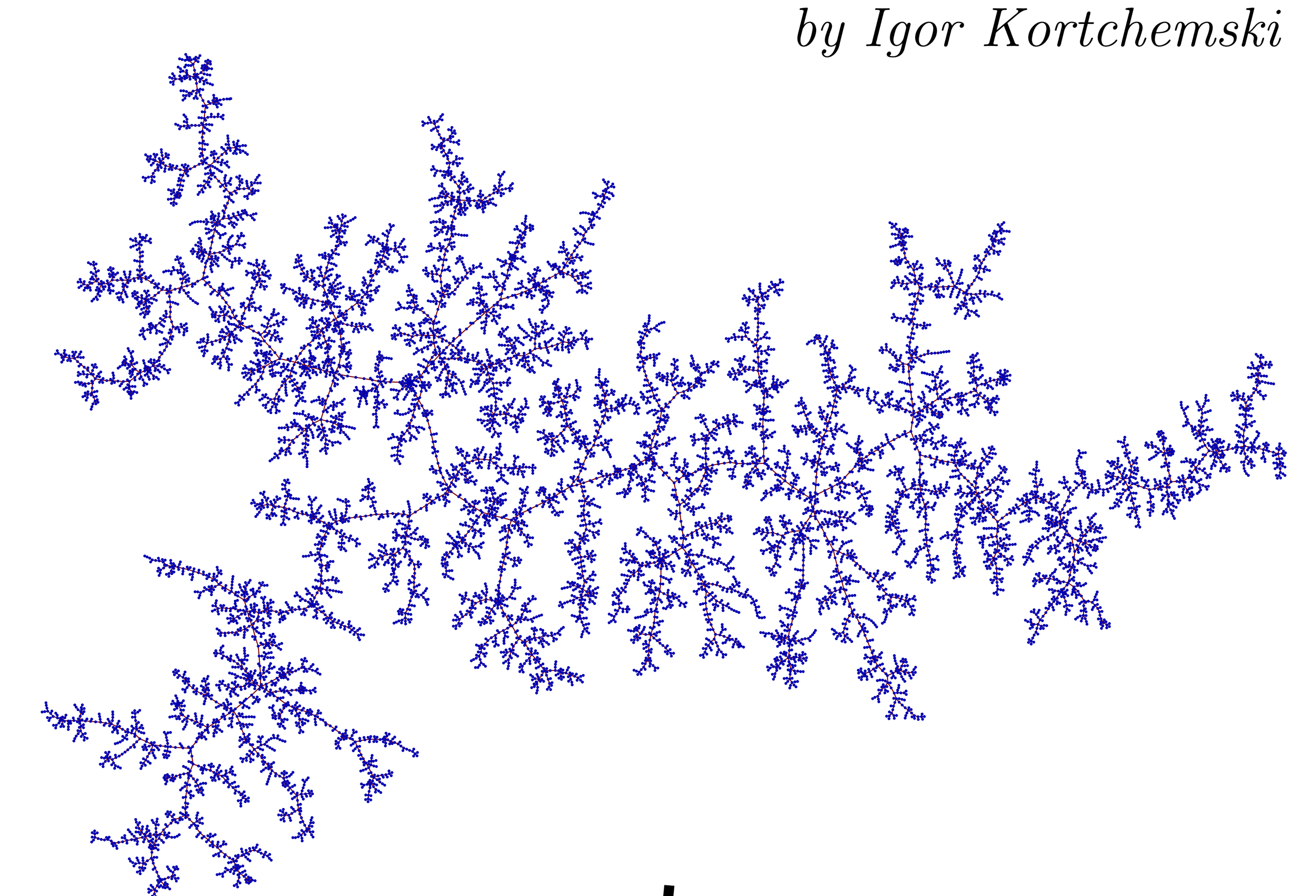


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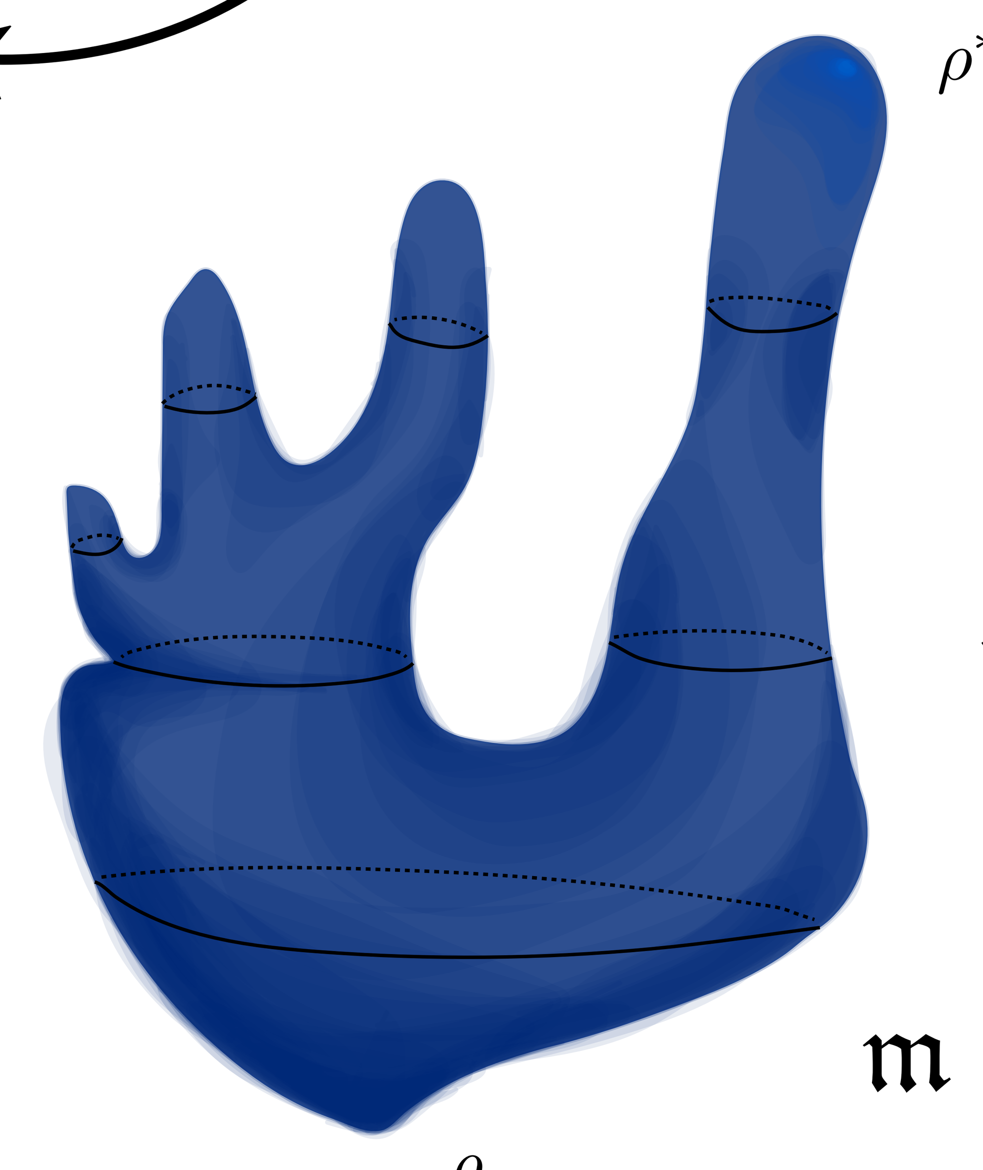



Brownian sphere

Thank you!

# MAT 933 Complex networks: theory and applications – Bovet

# MAT933 Complex Networks: Theory and Applications

Complex networks are used to model and analyse systems in a broad range of disciplines, including biology, sociology, economics, physics and information science.

The focus is on understanding **how components of a system are connected together and form complex patterns** that control the system's functions and dynamics. Studying these patterns allows one to **better understand and predict the behavior of these complex systems**.

Applications related to current problems in biological, urban, economic and social systems will serve as examples during the lecture.

Prior Knowledge: Basic notions of linear algebra, probability and some computational experience

Prof. Dr. Alexandre Bovet

English

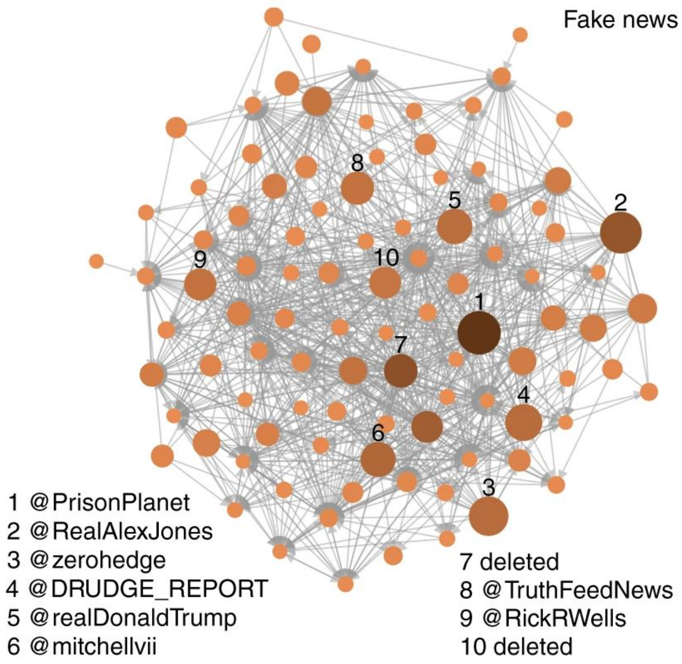
Lecture: Monday 10-12, Exercises: Thursdays 15-17

6 ECTS

# What are complex networks?

What are **networks**?

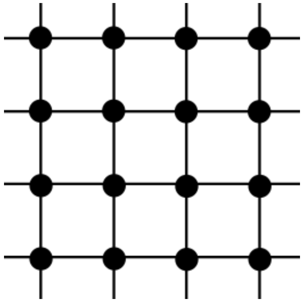
- System made of **nodes** (or vertices) and **links** (or edges)
- Framework to study the **connectedness** of real world systems
- Mathematically: a **graph** $G = (V,E)$
  - $V$: set of vertices
  - $E$: set of edges
- Networks are **models**



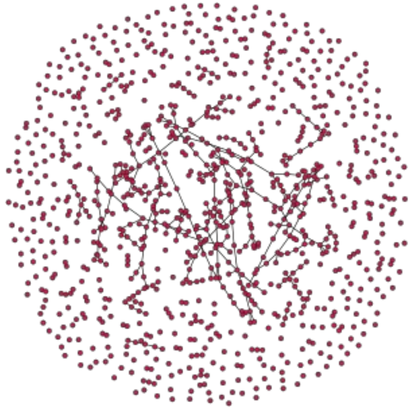
# What are complex networks?

Why **complex**?

- Complex networks are **not regular nor completely random**
- Complex networks are used to study and model **complex systems**



lattice



Erdős–Rényi random graph

# Complex systems

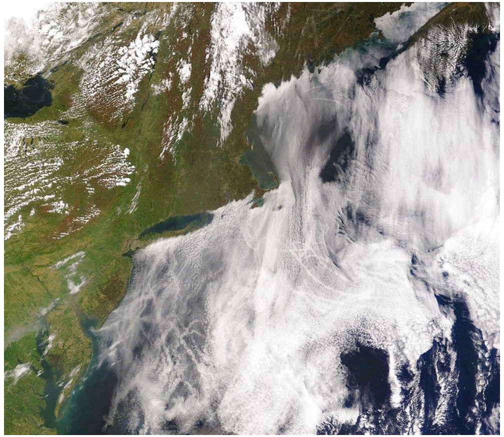
“The whole is more than the sum of the parts”

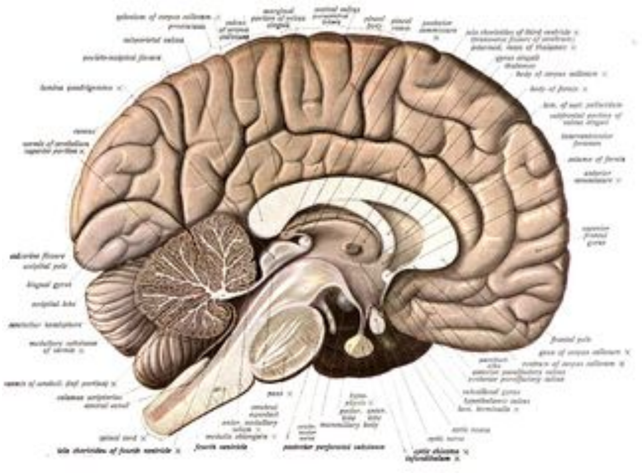
Aristotle misquote

“Complex systems consist of a large number of interacting **components**. The **interactions** give rise to **emergent hierarchical structures**. The components of the system and properties at systems level typically **change with time**.”

H. J. Jensen, *Encyclopedia of Complexity and Systems Science*

The patterns of interactions of complex systems can be represented by **networks**





# Objectives

Develop an intuition about networks (requires many skills: graph theory, linear algebra, probability, statistics, computation, dynamics, …)

Learn how to:

- Characterize the structural properties of networks
- Test the importance of structural factors
- Understand the mechanisms generating networks
- Characterize the dynamics of processes evolving on networks
- Gain a knowledge about applications of network science in several fields
- Learn to read and write scientific texts

# Plan

1. Introduction + Basic notions
2. Basic structural properties
3. Models of networks
4. Community structure
5. Dynamics and time scales
6. Random walks and community detection
7. Epidemics on networks
8. Percolation and robustness
9. Special topics (temporal networks, statistics and measurement errors, social networks, biological networks, urban networks, …)

# Exercises

**Mix of:**

- Problems to solve (pen and paper)
- Questions to answer using a computer (simulations)
  - Jupyter notebooks with python
- Papers to read and summarize
- Mini-project
  - Smaller version of the final project
  - Last two weeks
  - Must be done in groups of 2 to 3

Need 50% of the points across the semester to participate in the examination

# Examination

**Project**

- A broad topic will be given
- Write a report on a specific subtopic within the given topic in the form of a scientific article (abstract, significance, results, discussion, references, …)
- The report must contain some numerical simulations, a discussion of modeling issues and empirical data
- You must clearly indicate which ideas are new and which come from other resources
- The report does not need to contain original research results, but you must use some original research papers as resources (not just books and review articles)

**Timeframe**

- The topic will be given at the end of the semester
- Deadline for handing in the report end of January 2025

# MAT 959 Seminar in Data Science and Mathematical Modeling - Bovet

# MAT959: Seminar in Data Science and Mathematical Modeling

Alexandre Bovet, Delia Coculescu, Reinhard Furrer, Ashkan Nikeghbali

This seminar combines **research talks by international experts** on various topics in data science and mathematical modeling **and student presentations**.

The topics covered include applied statistics and probability, mathematical finance, risk theory, data science, machine learning, network science, and computational social sciences.

**Time:** Thursdays at 12:15

**Prior Knowledge**: Some background in data science and mathematical modeling, e.g. having successfully passed at least one of MAT519 Introduction to Mathematical Finance, MAT933 Complex Networks, or STA330 Modelling Dependent Data is suggested.

# Invited speakers from last semester

22.02.2024- **Huyên Xuan Pham** (Université Paris Diderot)
*Nonparametric generative modeling for time series via Schrödinger bridge*

07.03.2024 - **Ivo Sbalzarini** (Max Planck Institute of Molecular Cell Biology and Genetics)
*Numerical solution and data-driven inference of active material models in biology*

14.03.2024 - **Giona Casiraghi** (ETH Zürich)
*From biased urns to temporal networks: modeling multi-edge graphs with the generalized hypergeometric ensemble*

21.03.2024 - **Matteo Cinelli** (Sapienza University of Rome)
*Echo Chambers and Polarization in Online Social Media*

11.04.2024 - **Vaiva Vasiliauskaite** (ETH Zürich)
*A framework for approximating complex network dynamics with graph neural networks and identifying the limits of model's generalisation*

02.05.2024 - **Linna Du** (Swiss Re)
*A Mathematician's journey outside academia*

16.05.2024 - **Yucheng Yang**, Department of Banking and Finance UZH
*DeepHAM: A Global Solution Method for Heterogeneous Agent Models with Aggregate Shocks*

Updated list and abstracts are on the website: https://www.math.uzh.ch/fs24/mat959

# Required for 1 ECTS

For at least 70% of the seminars (9 for a typical semester):

- Actively participate in person (ask questions)
- Hand in a report

**Registration**: usual seminar booking

# Required for 3 ECTS

- Actively participate in person in at least 70% of the seminars
- Present one seminar (approximately 90min)
  - The presentation can be done with slides or on the blackboard
- Submit a written report not later than 2 weeks after the last seminar
  - 8-12 pages summarizing your seminar presentation.

The workload and grading scheme (pass/fail) are similar to a classical Seminar in Mathematics

We will comment on your presentation and report

**Registration**: Google form (link on the seminar website/course catalogue)

# Student seminars (3 ECTS)

**Example of topics:**

- Machine Learning on Graphs/Graph Signal Processing
- Machine Learning in Finance
- Numerical Methods in Finance
- Stochastic Block Model Inference in Networks
- Gaussian Markov Random Fields (GMRFs) and Gaussian processes on Graph

Other topics possible in function of the students background

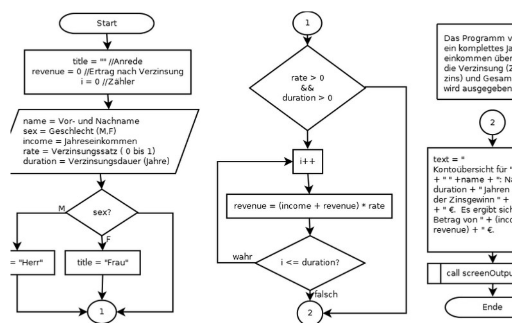
University of Zurich[UZH]

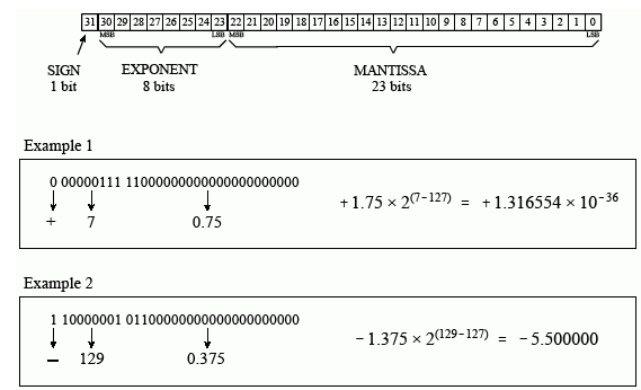
# Statistik Module

University of Zurich[UZH]

# STA222 Computers and Computing

## “Dry run” von M3L101 Computers and Computing




Vorkenntnisse: Python von Vorteil.

Prof. Dr. Reinhard Furrer

Englisch, V: Montag 13-15, U: Dienstag 8-10, 6 ECTS

University of Zurich[UZH]

# STA222 Computers and Computing

- What are computers?
- Digital Logic and Binary Representation
- Turing machine, computer program
- Operating systems, Unix/Linux, terminal, file systems
- Scripting and command line programs
- Floating point operations and representations
- Random numbers, random number generators,
- Compilers and programs, debugging and profiling
- Basic algorithms, algorithmic complexity
- Software and software IDE, repositories, software development
- Security, networks and protocols, ssh, authentification,
- Configuring and installing a server

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# STA121 Statistical Modeling

- Auswahl von modernen statistischen Methoden
- Focus auf Implementation und Interpretation

Vorkenntnisse: MAT901/MAT903 oder STA402, R

Dr. Zofia Baranczuk

Englisch, Montag 10-13 (2+1), 5 ECTS

University of Zurich[UZH]

# STA472 Good Statistical Practice

- ethical principles of statistical practice
- good written, visual & oral communication
- computational practice & computational efficiency
- workflow, git, LaTeX, simulation setup, ...

Limited spaces!

Flipped Classroom

Drs. E. Furrer, R. Furrer    4 ECTS,  Montag 4-6

# MAT 820 Numerisches Praktikum - Sauter
# MAT 828 Numerical methods of ODEs

# MAT784 Seminar on Knot Theory – Wildi

# Online Presentations

- MAT 562 Seminar Frobenius Algebras and 2-Dimensional TQFT – Bobtcheva

- MAT 782 Lecture Spin Geometry – Jiang