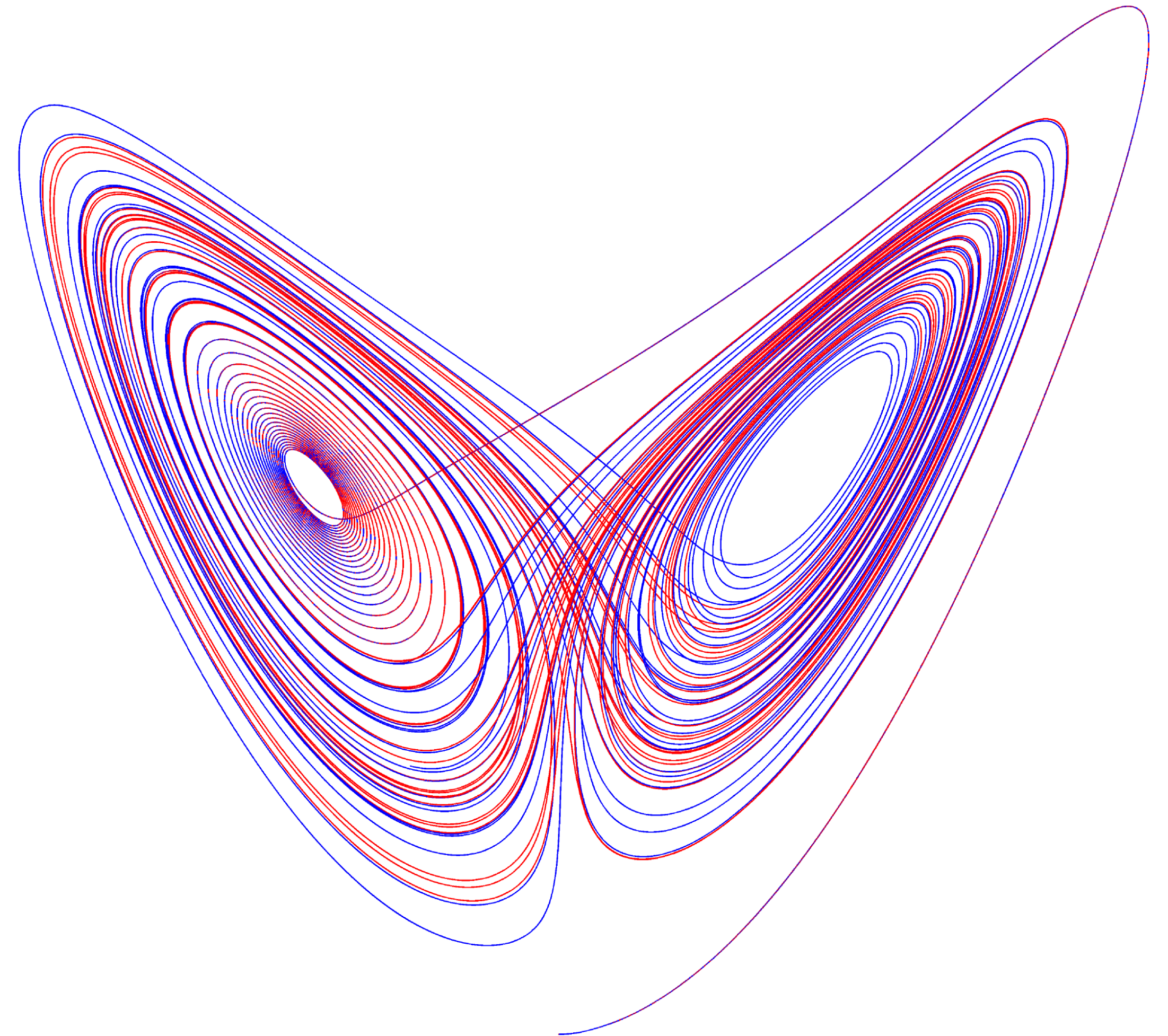


Seminar: Introduction to chaotic dynamical systems

Sophie Schmidhuber



**Universität
Zürich** ^{UZH}

What is a dynamical system?

A photograph of a stock market ticker board, representing a dynamical system like financial markets.

IBM	221.90	●	-3.30	-1.5%	114.81
MSFT	112.56	●	-2.57	-2.2%	3879.65
AMZN	3716.78	●	-148.50	-3.8%	298.69
GOOGL	282.04	●	-17.08	-5.7%	96.75
FB	93.57	●	-3.19	-3.3%	91.17
APPL	89.93	●	-1.03	-1.1%	1077.38
DIS	1019.23	●	-57.62	-5.4%	1503.79
BA	1417.56	●	-40.92	-2.7%	72.96
WMT	71.54	●	-1.43	-2.0%	3417.92
ORCL	3320.78	●	-105.40	-3.1%	137.21
INTC	133.27	●	-4.14	-3.0%	5774.98
CRM	1648.86	●	-128.45	-2.2%	63.54
ADBE	2317.48	●	-115.56	-4.4%	2622.00



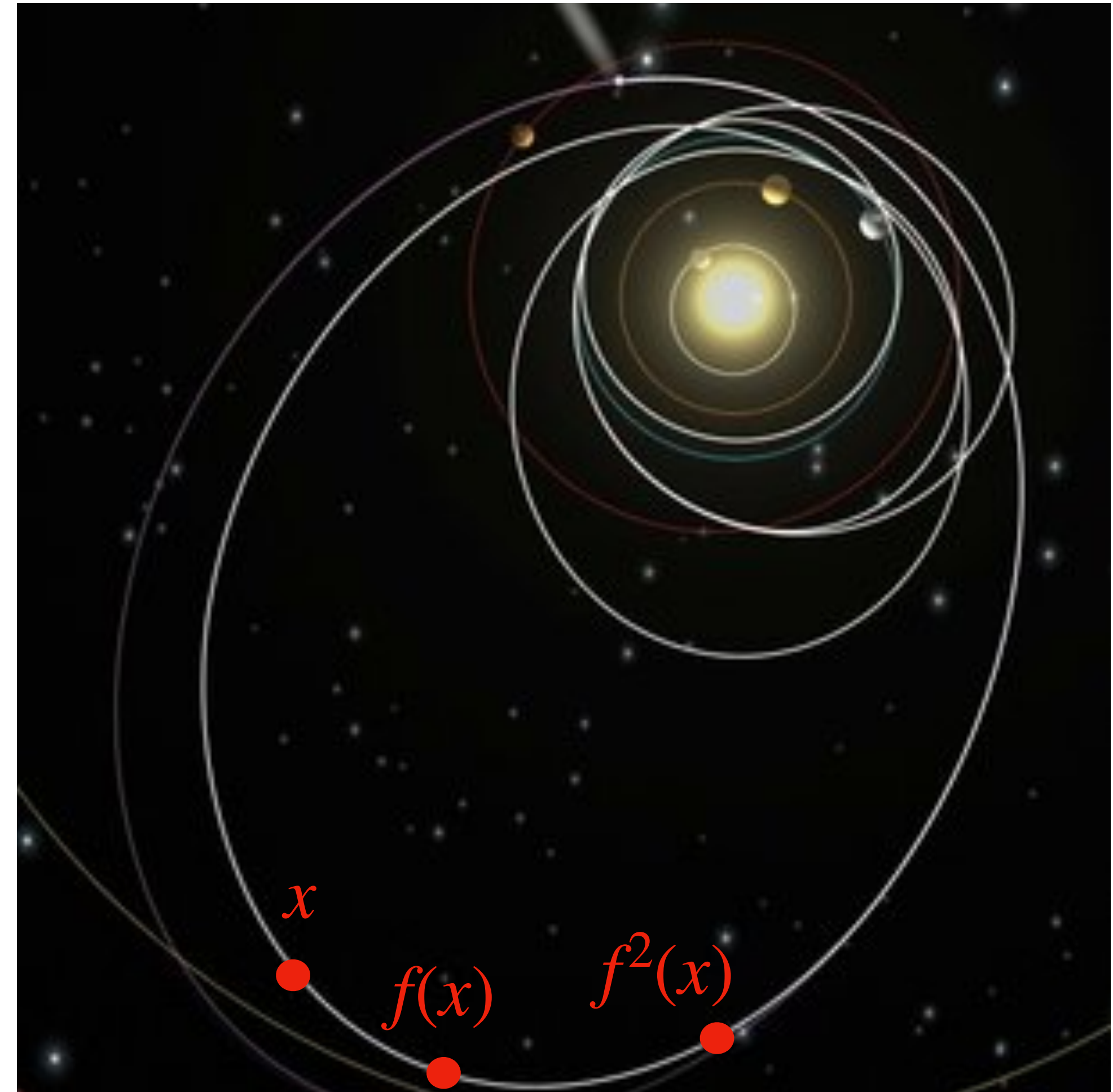
Guiding question: Where do points go and what do they do when they get here?

More formally...

Let $f: X \rightarrow X$ be a function from a space X to itself. We define the orbit of a point $x \in X$ as:

$$O^+(x) = \{x, f(x), f^2(x), \dots\}$$

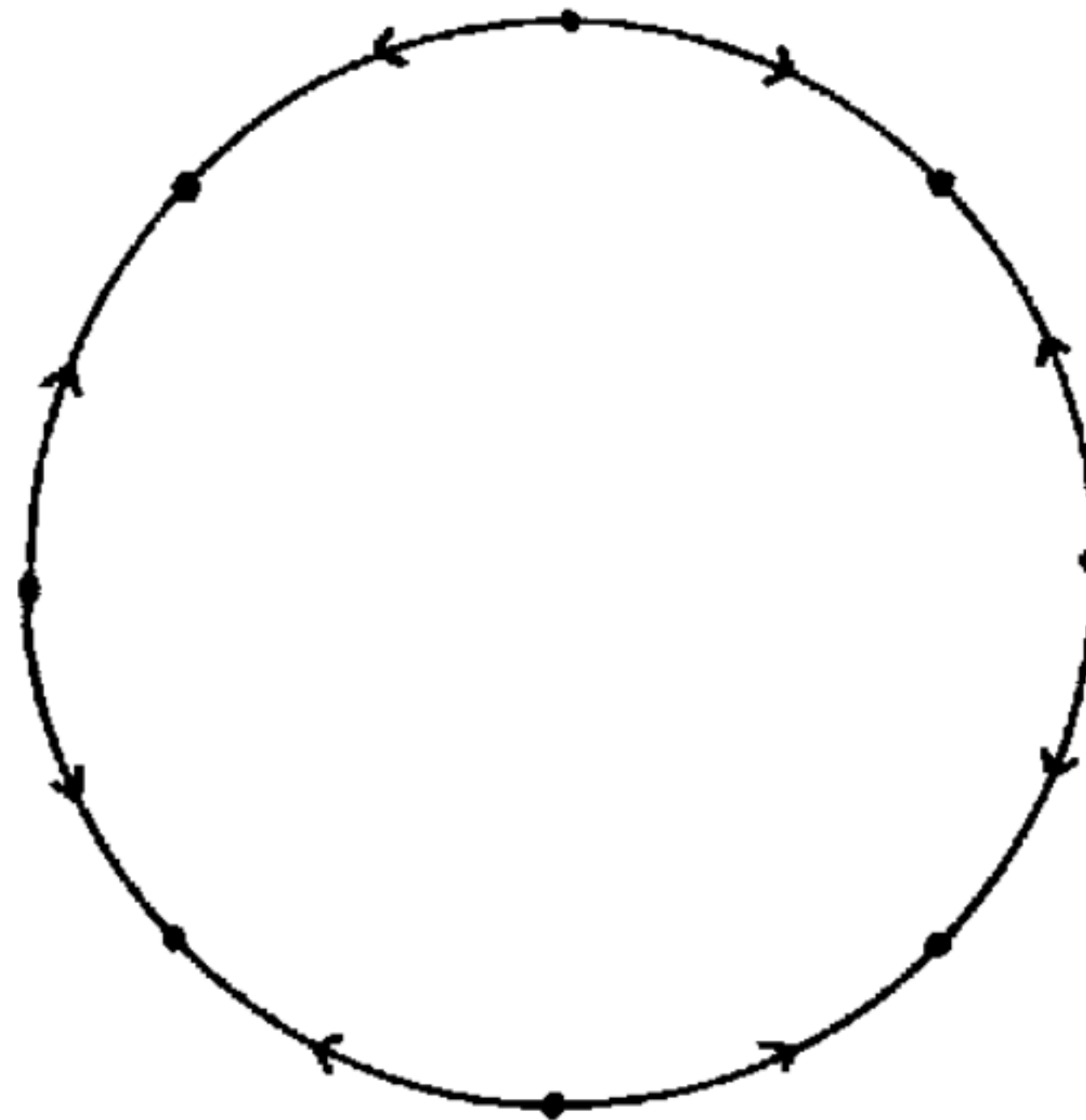
Guiding question: What is the asymptotic behavior of orbits?



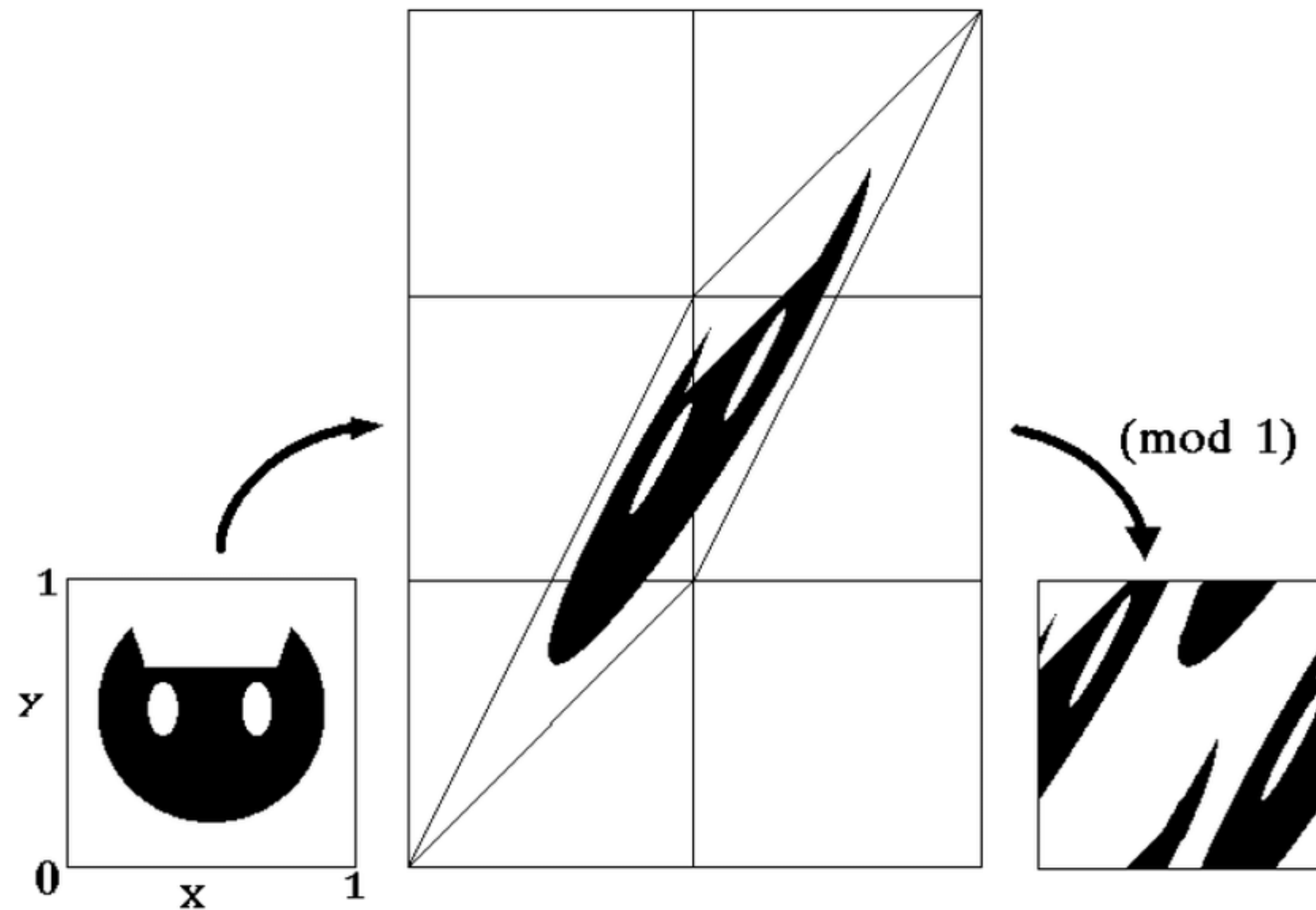
Example: Rotations on the circle

Theorem. Let $\lambda \in \mathbb{R}$ and consider the map $T_\lambda(\theta) = \theta + 2\pi\lambda$ on the circle. Then

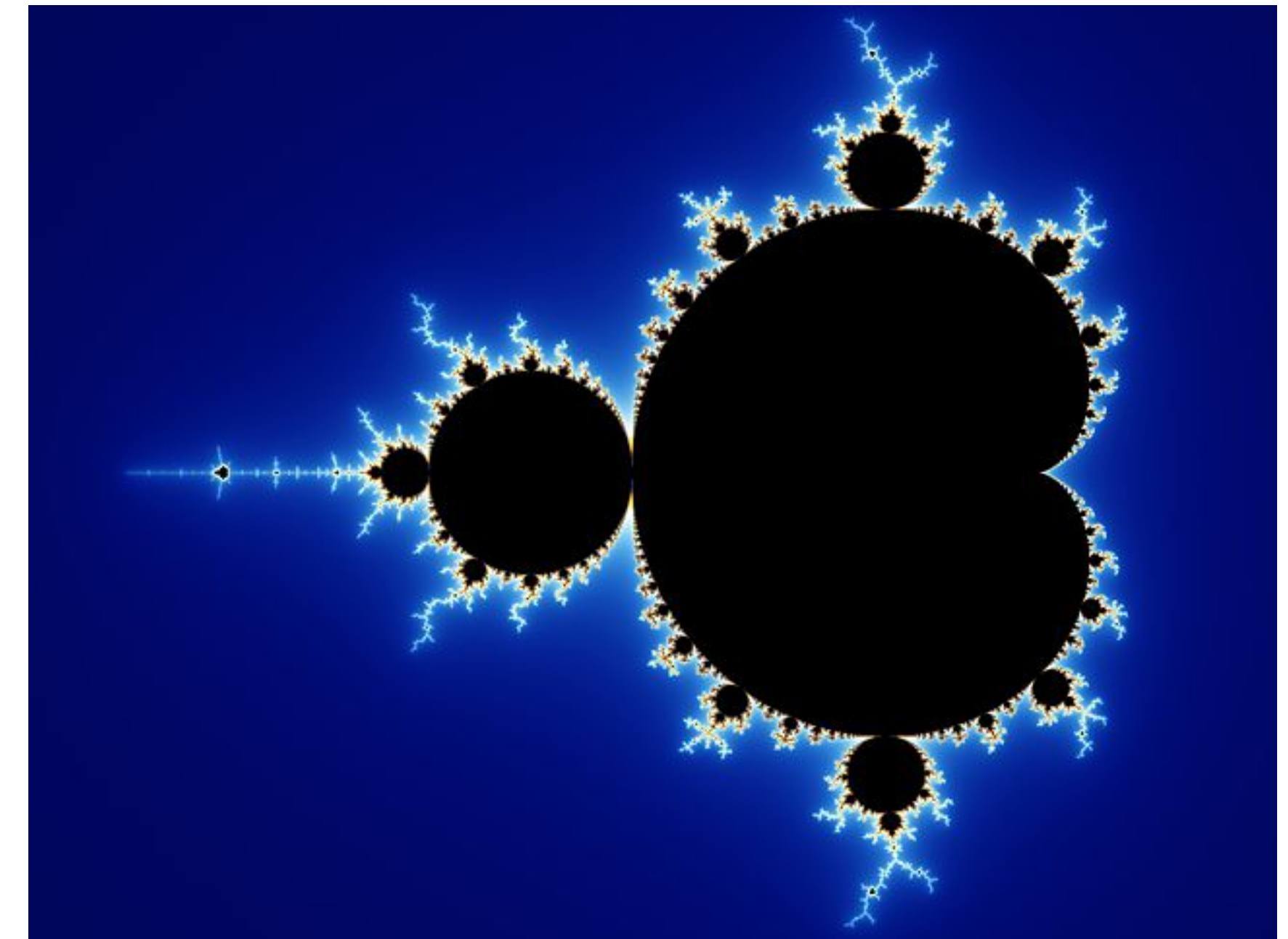
- if λ is rational, all orbits are periodic,
- if λ is irrational all orbits are dense.



But there exist even more interesting systems...



The cat map
(Higher dimensional dynamics)



The Mandelbrot set
(Complex analytic dynamics)

Main reference

*An introduction to
Chaotic dynamical systems*

By Robert L. Devaney

An Introduction to
**Chaotic Dynamical
Systems** Second Edition

Robert L. Devaney

