Seminar: Introduction to chaotic dynamical systems

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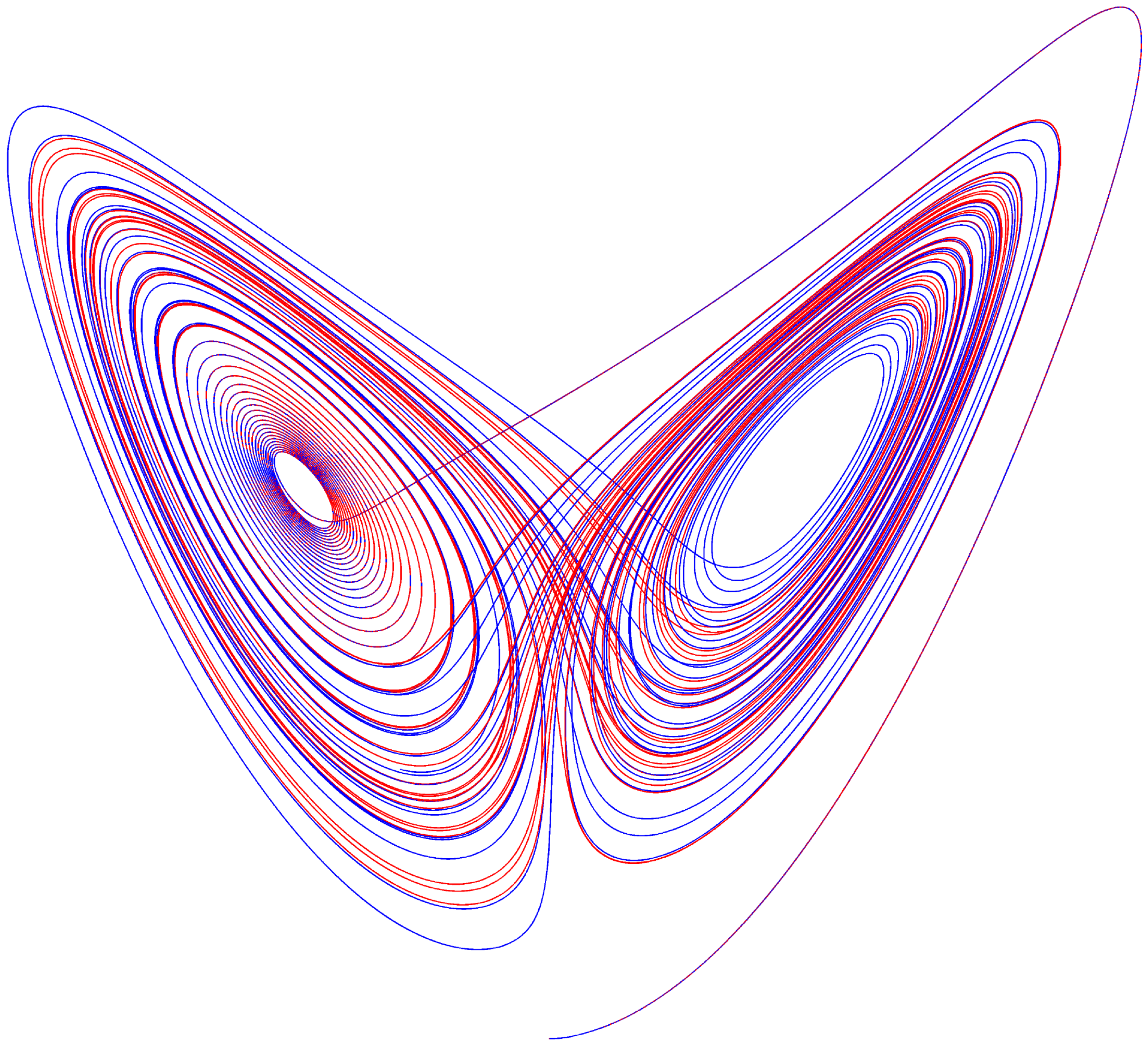

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What is a dynamical system?

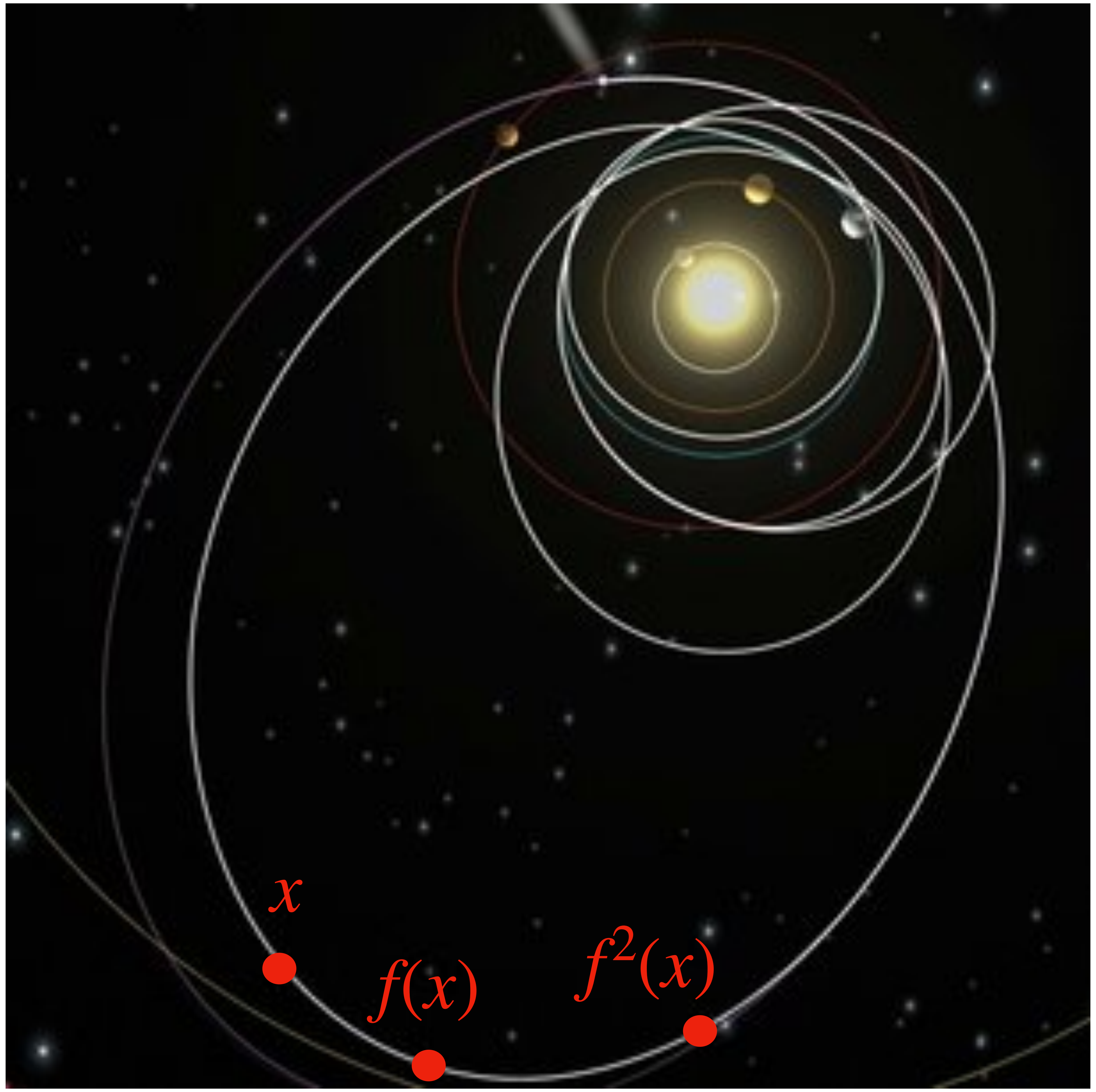
Guiding question: Where do points go and what do they do when they get here?

More formally…

Let $f : X \to X$ be a function from a space X to itself. We define the orbit of a point $x \in X$ as:

$$O^+(x) = \{x, f(x), f^2(x), \dots\}$$

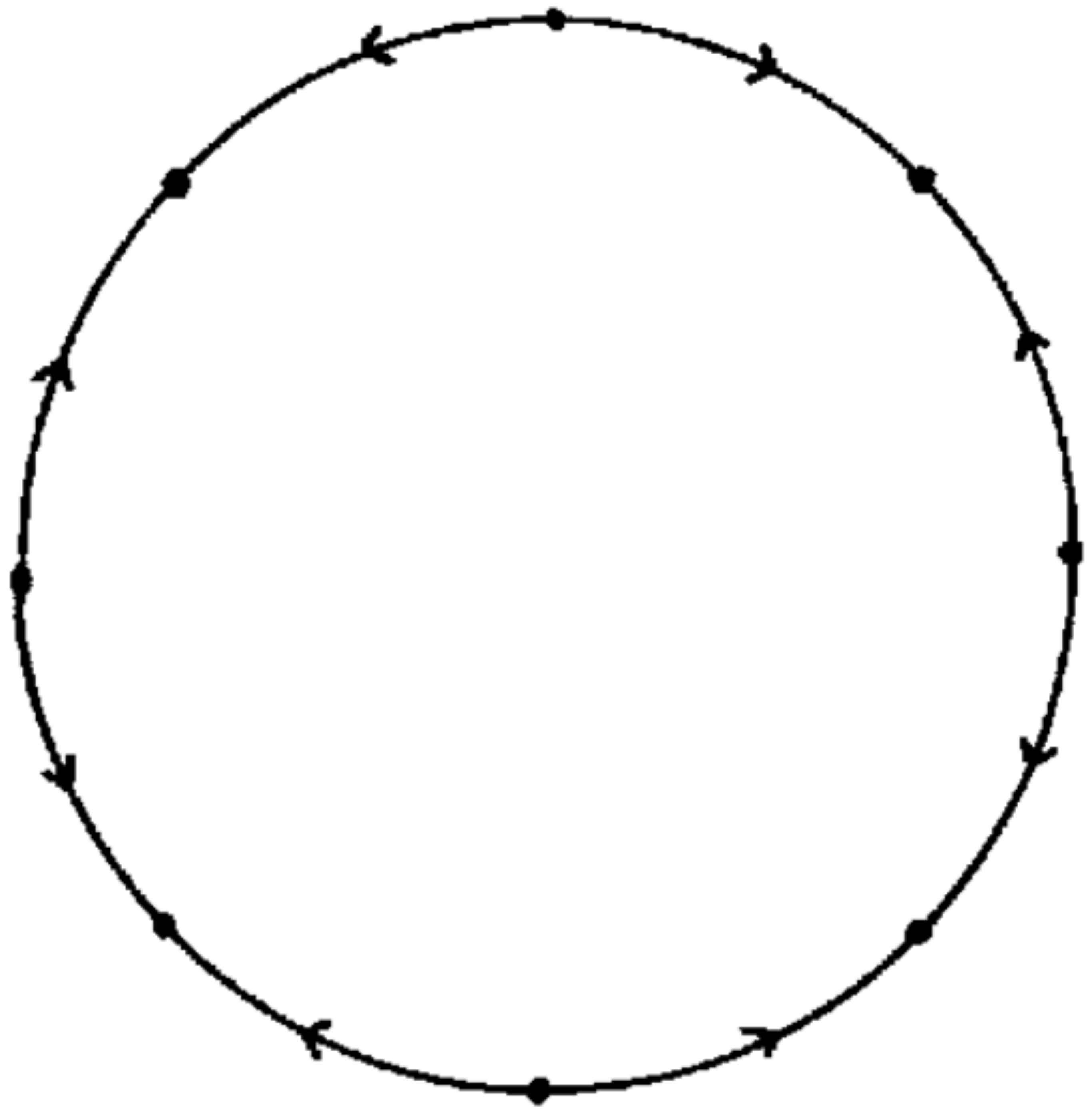
Guiding question: What is the asymptotic behavior of orbits?



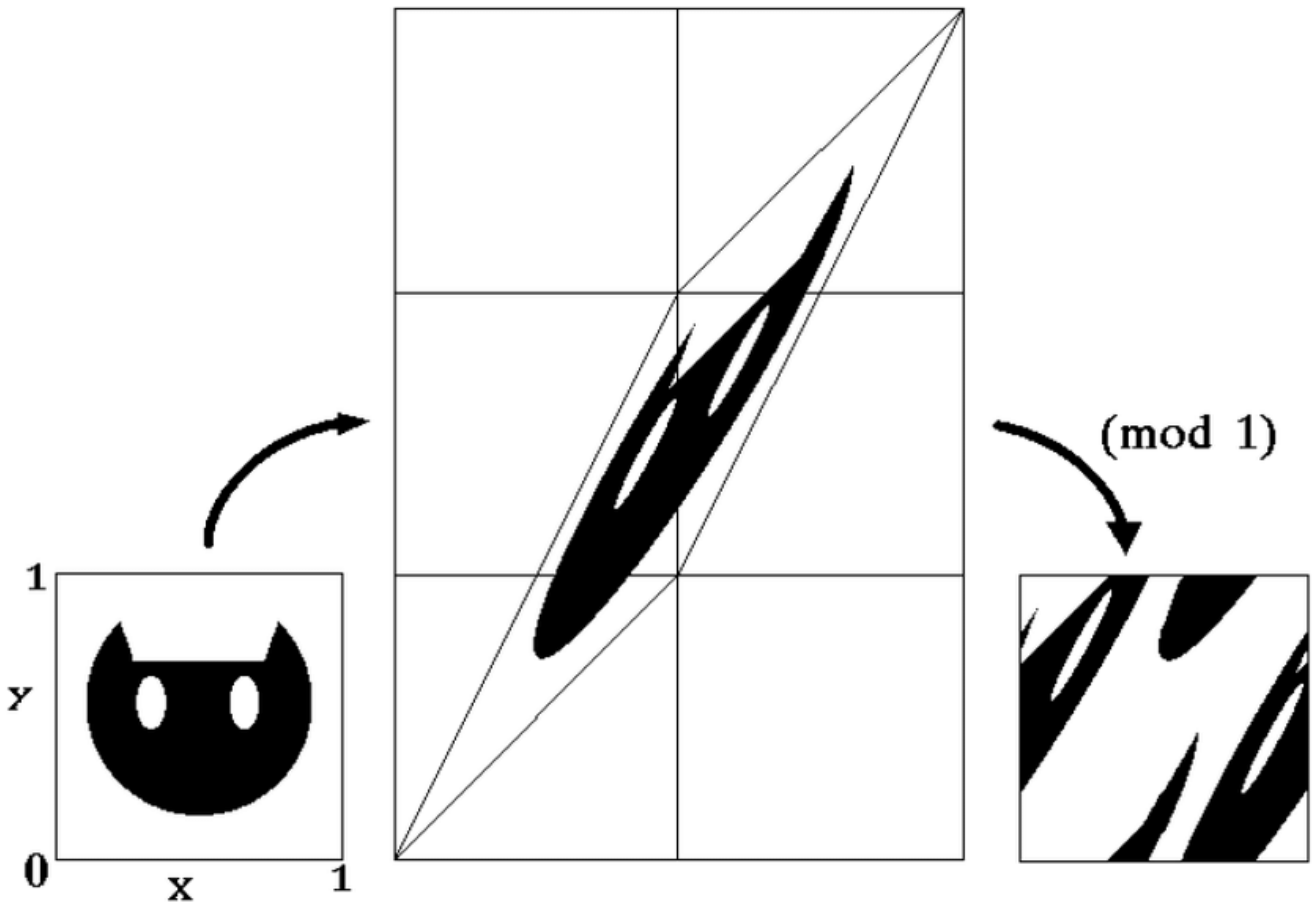
Example: Rotations on the circle

Theorem. Let $\lambda \in \mathbb{R}$ and consider the map $T_\lambda(\theta) = \theta + 2\pi\lambda$ on the circle. Then

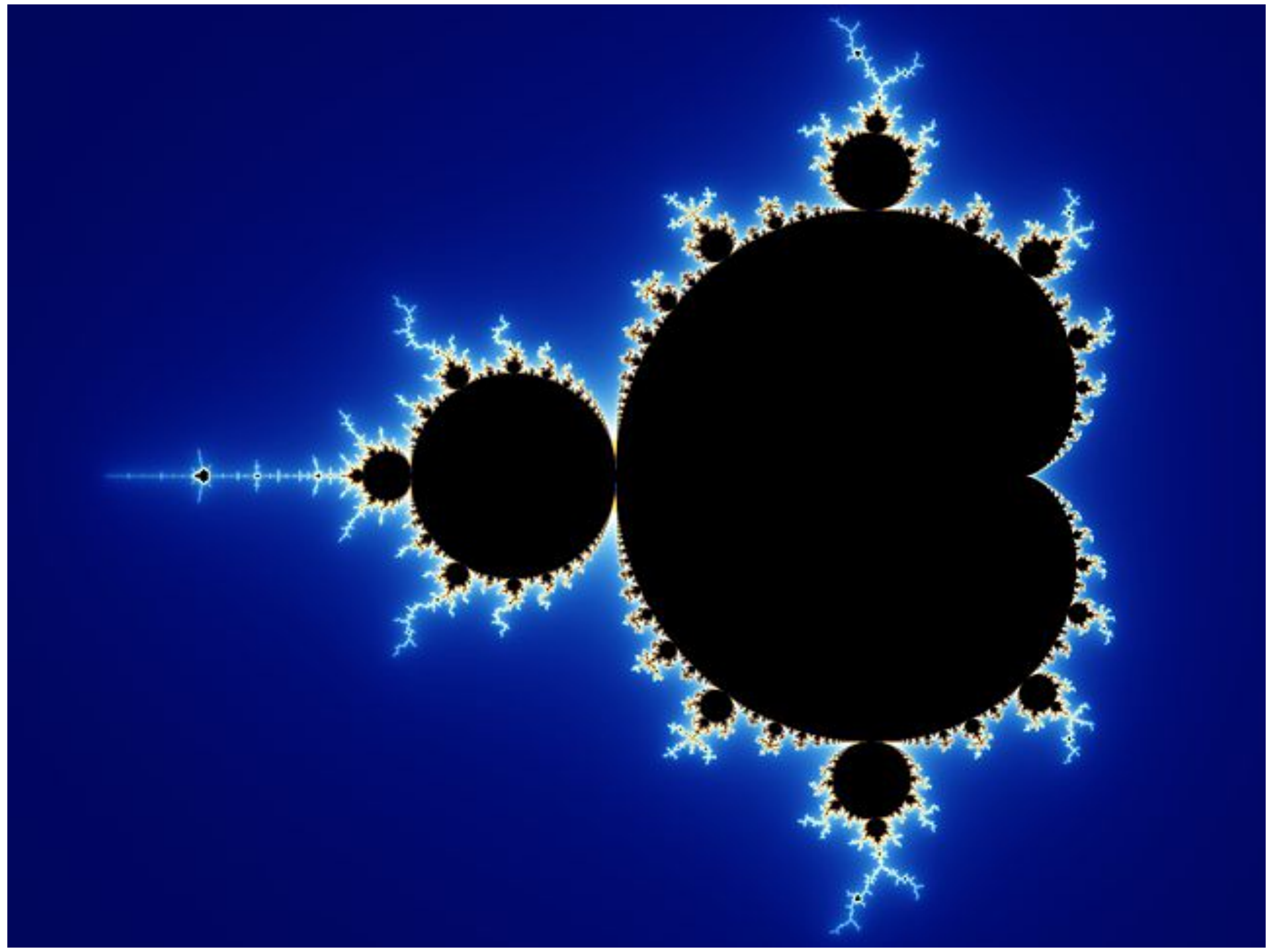
- if λ is rational, all orbits are periodic,
- if λ is irrational all orbits are dense.



But there exist even more interesting systems…



The cat map
(Higher dimensional dynamics)



The Mandelbrot set
(Complex analytic dynamics)

Main reference

An introduction to
Chaotic dynamical systems

By Robert L. Devaney