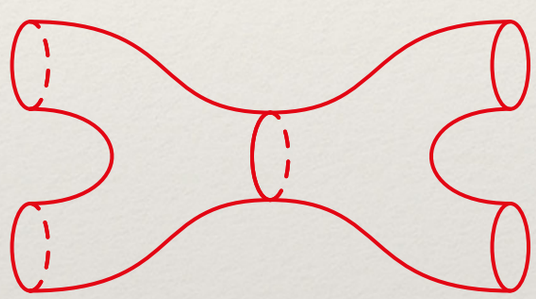
Ivelina Bobtcheva

Student seminar

Frobenius algebras and 2-dimensional Topological Quantum Field Theories

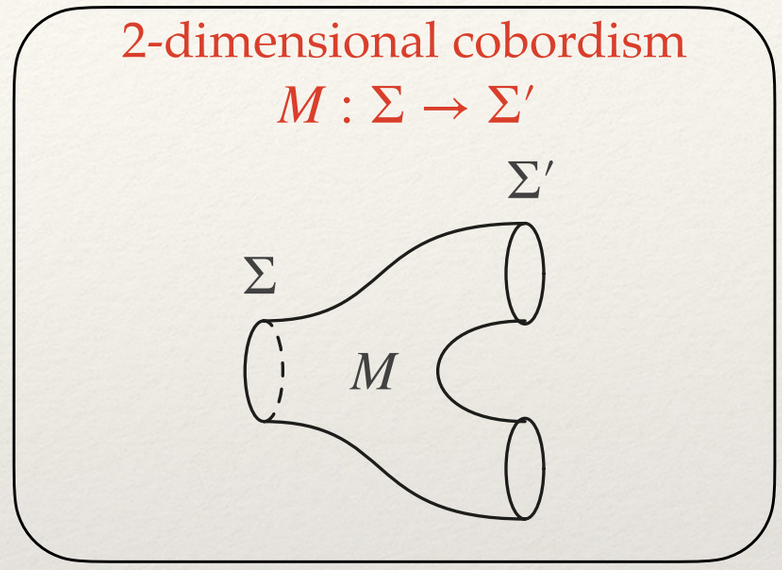


What is a Frobenius Algebra?

A Frobenius algebra A is a finite-dimensional algebra equipped with **nondegenerate bilinear form** which satisfies a certain compatibility axioms with respect to the multiplication. Examples are **matrix rings, group rings** and **the ring of characters of a representation**.

What is a Topological Quantum Field Theory (TQFT) ?

An ***n*-dimensional TQFT** is a functor F from the category of n-dimensional cobordisms to the category of vectors spaces **Vect** which associates to each manifold Σ of dimension $n-1$ a vector space $F(\Sigma)$, and to each n-manifold M whose 'in-boundary is Σ and whose 'out-boundary' is Σ', a linear map between $F(\Sigma)$ and $F(\Sigma')$.



How are Frobenius algebras related to TQFT's?

We will show that the category of 2-dimensional TQFT's is equivalent to the category of commutative Frobenius Algebras.

Why do we care?

The result shows how the study of a purely topological category can be reduced to the study of a certain algebraic structure. Extending it to higher dimensional spaces is an object of an active research.

Description

The seminar course offers an introduction to the topic by studying the TQFT's on the category of cobordisms 2-dimensional oriented manifolds (surfaces) and its equivalence with the symmetric monoidal category freely generated by a Frobenius algebra object.

Topics

- The category of cobordisms of oriented surfaces.
- Frobenius algebras and bialgebras
- Monoids and monoidal categories
- Proof of the equivalence theorem
- Invariants produced by specific examples of Frobenius algebras.
- Other cobordism categories (homotopy TQFT's, TQFT's of 2-dimensional CW-complexes)

Course material

- Joachim Kock, Frobenius Algebras and 2-D Topological Quantum Field Theories (London Mathematical Society Student Texts Book 59)
- Frank Quinn, Lectures on axiomatic topological quantum field theory. In Geometry and quantum field theory (Park City, UT, 1991), pp. 323–453. Amer. Math. Soc., Providence, RI, 1995

Practical information

- Prerequisites: Linear algebra I, Linear algebra II, Algebra, Topology
- You will be expected to prepare a talk and present it in class, hand in a typed (LaTex) version of the talk and participate in discussions during the seminars