Homework 7

1. A polyhedron is called regular is every face has the same number of edges, and every vertex meets the same number of edges.

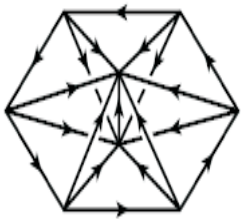
   (a) Let P be a convex regular polyhedron such that each face has p edges and every vertex meets q edges. Prove that

   $$\frac{1}{p} + \frac{1}{q} = \frac{1}{2} + \frac{1}{e}$$

   where e is the number of edges.

   (b) Prove that there are only five convex regular polyhedra.

2. Construct a 3-dimensional simplicial complex from n tetrahedra (3-simplices) $T_1, \ldots, T_n$ by the following two steps. First arrange the tetrahedra in a cyclic pattern as in the figure, so that each T_i shares a common vertical face with its two neighbors T_{i-1} and T_{i+1} (where subscripts are interpreted modulo n). Then identify the bottom face of T_i with the top face of T_{i+1} for each i (again, subscripts are interpreted modulo n). Show that the simplicial homology groups of X in dimensions 0, 1, 2, 3 are $\mathbb{Z}$, $\mathbb{Z}/n$, 0, $\mathbb{Z}$ respectively. (This space is an example of a lens space.)



3. Compute degree of the following maps of the sphere S^n:

   (a) $f : S^n \to S^n$ is a reflection with respect to a hyperplane through the origin.

   (b) $f(x) = -x$.

   (Hint: Construct convenient triangularization of the sphere. You may use that degree is independent of triangularization.)