Homework 6

1. If $k < m, n$, show that any map $S^k \to S^m$ is null homotopic, and that the same is true for any map $S^k \to S^m \times S^n$.

2. Let X_1 and X_2 be topological spaces that admit triangulation. Prove that the product space $X_1 \times X_2$ admits a triangulation too.

3. Construct a triangulation for the dunce hat (see Problem Set 3) and using this triangulation compute its fundamental group.

4. Let P be a union of finitely many convex polyhedrons P_i in $\mathbb{R}^d$ such that for any i, j, k, we have $P_i \cap P_j \cap P_k \neq \emptyset$. Prove that P is simply connected.

5. Compute the fundamental group for the following spaces:

   (a) the quotient space of S^n obtained by identifying the north and the south poles,

   (b) the space obtained by identifying the circle $S^1 \times \{x_0\}$ of the torus $S^1 \times S^1$ and the equator circle of the sphere S^2.

   (c) the space obtained from two tori $S^1 \times S^1$ by identifying the circles $S^1 \times \{x_0\}$.