Homework 7

1. (a) Show that the connected sum of a projective plane with itself is homeomorphic to a Klein bottle.

   (b) Show that a connected sum of the Klein bottle and a projective plane is homeomorphic to a connected sum of a torus and a projective plane.

2. Let $h : |K| \to S$ be a triangulation of a compact surface. Prove that K has dimension 2, every edge of K lies in precisely two triangles, the triangles of K that contain a particular vertex fit together in a cone (see Armstrong, Problem 7, page 158).

3. Show that the surface corresponding to the word

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1}$$

   is homeomorphic to a connected sum of g tori, and the surface given by the word

$$(a_1 a_1) \cdots (a_g a_g)$$

   is homeomorphic to a connected sum of g projective planes.

4. Determine what surfaces are represented by the words

$$a_1 \cdots a_g a_1^{-1} \cdots a_g^{-1} \quad \text{and} \quad a_1 \cdots a_g a_g \cdots a_1.$$