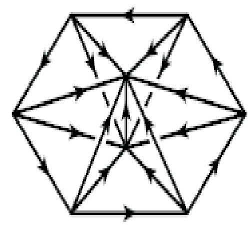
Homework 8

1. Calculate the homology groups of a triangulation of the sphere S^2 with k holes.

2. Construct a 3-dimensional simplicial complex from n tetrahedra (3-simplices) $T_1, \ldots, T_n$ by the following two steps. First arrange the tetrahedra in a cyclic pattern as in the figure, so that each T_i shares a common vertical face with its two neighbors T_{i-1} and T_{i+1} (where subscripts are interpreted modulo n). Then identify the bottom face of T_i with the top face of T_{i+1} for each i (again, subscripts are interpreted modulo n). Show that the simplicial homology groups of X in dimensions 0, 1, 2, 3 are $\mathbb{Z}$, $\mathbb{Z}/n$, 0, $\mathbb{Z}$ respectively. (This space is an example of a lens space.)



3. (weak Perron-Frobeneous theorem) Let A be a nondegenerate $(n \times n)$-matrix with nonnegative entries. Using the Brouwer theorem, prove that A has a positive eigenvalue with eigenvector whose coordinates are nonnegative.

4. Let K and L be finite simplicial complexes. By triangulating $|K| \times |L|$ appropriately, show that

$$\chi(|K| \times |L|) = \chi(|K|) \cdot \chi(|L|).$$