

Homework set 6 (due Sat., March 11)

1. Let $\phi_t : TM \rightarrow TM$ be the geodesic flow. Prove that ϕ_t is an isometry of the Sasaki metric on TM for every t if and only if M has constant curvature equal to 1.
2. Compute the Liouville measures for the unit tangent bundle $T^1\mathbb{H}^2$.
3. Let Γ be a totally discontinuous subgroup of $\mathrm{PSL}(2, \mathbb{R})$ and $M = \Gamma \backslash \mathbb{H}^2$. For $\gamma \in \Gamma$, we denote by $|\lambda_\gamma|$ the maximum of the absolute values of the eigenvalues of γ . Prove that the set of lengths of closed geodesics in M is given by

$$\{2 \log |\lambda_\gamma| : \gamma \in \Gamma\} - \{0\}$$

Note that in this problem we allow closed geodesics that go along the same loop multiple times.