

MATH 425 REVIEW 2 (CH.4–5)

1 Basic Concepts

1.1 Discrete random variables (r.v.'s)

- Discrete r.v.'s, expected value, variation.
- Important r.v.'s:
 - binomial;
 - Poisson;
 - geometric;
 - negative binomial;
 - hypergeometric.

Test questions:

1. What is the expected value of a r.v. X such that $P(X = a) = P(X = -a)$?
2. What is the relation between binomial and Poisson r.v.'s?
3. Match the following quantities with the r.v.'s that you know (what assumptions are you making?):
 - (a) the number of cancer patients in US;
 - (b) the number of times one hits the target during one-hour shooting practice;
 - (c) the number of calls to randomly chosen phone numbers until you reach a person with the name John Smith;
 - (d) the number of calls a telemarketer makes to sell a product to 10 people;
 - (e) the number of defective items in a sample of fixed size that a factory produced.

1.2 Continuous r.v.'s

- Continuous r.v., probability density function, cumulative distribution function, expectation, variance.
- Important r.v.'s:
 - uniform;

- normal;
- exponential.

Test questions:

1. Suppose that for a r.v. X , $E[X] = a$. Is it always true that $P(X < a) = P(X > a)$?
2. What is the relation between binomial and normal r.v.'s?

2 Problems

1. Ch.4 self-test problems: #3,6,11,15.
2. Ch.5 self-test problems: #6,10,13,15.
3. During the second world war, the Nazi air force dropped 600 bombs on London. The map of London is divided into 500 blocks of equal size. Assume that every block has equal probability of being hit.

- (a) What is the probability that a given block is not hit?
- (b) What is the probability that a given block is hit more than once?

You should be able to solve this question using either binomial or Poisson random variables.

4. An airline estimates that 3% of people who make reservations don't show up. Because of this, the airline sells 102 tickets for 100-seat plane. We assume that the plane is fully booked.
 - (a) What is the probability that exactly 100 passengers will show up?
 - (b) What is the probability that all passengers that show up will be seated?
 - (c) The airline sells each ticket \$200, and if a passenger cannot be seated, he is paid \$300. Find the expected value of the revenue of the airline.
5. At time $t = 0$ hours, we install a lamp. We may use lamps produced by either company A or B . The lifetimes of the lamps of companies A and B are the exponential random variables with $\lambda = 1$ and $\lambda = 10$ respectively. It is equally likely that we choose a lamp from company A or company B .
 - (a) What is the probability density function of the lifetime of the lamp that was installed?
 - (b) What is the probability that the lamp functions for more than 50 hours?
 - (c) What is expected value of the time that the lamp will function?

- (d) Given that the lamp functions for more than 50 hours, what is the probability that the lamp from company A was installed?
6. We transmit a bit b of information (i.e., b is 0 or 1) over a channel with noise. The noise is a normal r.v. X with mean 0 and variance 0.25. This means that if we send a bit b , then on the other end, one receives the number $\tilde{b} = b + X$. We fix a number e in $(0, 0.5)$. If $|\tilde{b}| < e$, we interpret the message as 0; if $|\tilde{b} - 1| < e$, we interpret the message as 1; otherwise, we request to resend the message. In (a)–(e), we suppose that $e = 0.2$.
- (a) What is the probability that message is interpreted correctly based on the first transmission?
- (b) What is the probability that the message is interpreted incorrectly based on the first transmission?
- (c) What is the probability that it takes exactly 3 transmissions to interpret the message and the message is interpreted correctly?
- (d) What is the probability that the message is interpreted correctly?
- (e) What is the probability that it takes more than 10 submissions to interpret the message?
- (f) Choose e such that the probability of wrong interpretation in one transmission is less than 0.005. (You might need to use the table of the standard normal distribution.)
- (g) To improve the situation in (f), we send the bit *three* times, and interpret the message as 0 (or 1) if at least *two* of the transmissions are interpreted as 0 (or 1). Find the probability that the message is interpreted incorrectly.