MATH 425 REVIEW 3 (CH.6–8)

1 Basic Concepts

1.1 Joint distribution of rv's

- Double integrals (from Calculus III).
- Joint commutative distribution function (cdf) and joint probability density function (pdf).
- Multinomial distribution.
- Gamma distribution.
- Independent rv's.
- Sums of random variables, convolutions.
- Conditional cdf and pdf.
- Order statistics.
- Joint distribution of functions of rv's.

Test questions:

1. Represent the double integral $\int_{-1}^{1} \int_{0}^{1-x^2} f(x, y) dy dx$ as $\int_{?}^{?} \int_{?}^{?} f(x, y) dx dy$
2. Let (N_1, N_2) be multinomial distribution with parameters (n, p_1, p_2). Are N_1 and N_2 independent?
3. If X_1 and X_2 are independent normal random variables with parameters (μ_1, σ_1) and (μ_2, σ_2), what is the distribution of $X_1 + X_2$?
4. If X_1 and X_2 are independent exponential random variables with parameter λ, what is the distribution of $X_1 + X_2$?
5. Let $X_{(i)}$, $i = 1, \ldots, n$, be the order statistics of independent random variables X_i, $i = 1, \ldots, n$. Are $X_{(i)}$ and $X_{(j)}$ independent?

1.2 Expectation of sums of rv's

- Formulas for the expectation and variance of sum of random variables.
- Computations using 0-1 random variables.
- Covariance and correlations.
- Conditional expectation and variance.
- Computing expectation and variance by conditioning.
- Moment generating functions (mgf).

Test questions:

1. Is true that $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$?
2. If $M_X(t)$ is mgf of a rv X. What is $M_X(0)$?

1.3 Limit Theorems

- Markov and Chebyshev inequalities.
- Weak law of large numbers.
- Central limit theorem.

Test questions:

1. Which one is more reliable: Chebyshev inequality or the central limit theorem? Which one gives better estimates?

2 Problems

1. Ch.6: #3, 12, 14, 15.
2. Ch.7: #3, 8, 12, 14, 17, 18.
3. Ch.8: #5, 6, 8, 10.
4. Let X and Y be random variables such that the point is (X, Y) is uniformly distributed inside of the triangle with vertices $(1, 0)$, $(2, 0)$, $(1, 2)$. That is, the joint density function of X and Y is 1 in the interior of the triangle and 0 otherwise.
   (a) Find pdf of X and pdf of Y.
   (b) Are X and Y independent?

(c) Compute the covariance of X and Y.

(d) Let $U = 2X + Y$ and $V = X - Y$. Find the joint pdf of U and V.

5. A coin of diameter 3/4 in. is dropped on the table divided into squares with sides 1 in.

    (a) Introduce a sample space and random variables that describe this experiment.

    (b) Find the probability that the coin fall in the interior of one of the squares.

6. Do the previous problem when coin is replaces by a needle of length 0.5 in.

7. A rope of length 1 ft. is cut into 3 pieces at two places that are uniformly distributed on the rope. Find the expected value of the length of the smallest piece of the rope.

8. Suppose that each customer that comes to a restaurant orders an apple pie with probability 0.1.

    (a) If the number of customers that come to a restaurant during one day is Poisson rv with $\lambda = 200$, find the expected number of apple pies ordered during one day.

    (b) Suppose that the restaurant has exactly 200 customers on a given day. How many pies should the restaurant have to be sure that, with probability 95%, everybody is served?

    (c) If the number of customers is very large, what can you say about the percentage of customers that order apple pies?

    (d) How many customers should the restaurant have so that, with probability 95%, less than 11% of customers order apple pies?