Dynamical Systems and Ergodic Theory

MATH 36206 and MATH M6206

Lecturer: Professor Alexander Gorodnik <a.gorodnik@bristol.ac.uk> Howard House 5a

Course Webpage: https://people.maths.bris.ac.uk/~mazag/ds17/

Class Times: Monday 10-11am at MATH SM4
Wednesday 11-12pm at QUEENS BLDG 1.58
Friday 11-12pm at MATH SM4

Office hours: Friday 10-11am, Howard House 5a

Prerequisites: MATH10003 Analysis 1A, MATH10006 Analysis 1B, MATH11007 Calculus 1

Course Description: A dynamical system can be obtained by iterating a function or considering evolutions of physical systems in time. Even if the rules of evolution are deterministic, the long term behaviour of the systems are often unpredictable and chaotic. The Theory of Dynamical Systems provides tools to analyse this chaotic behaviour and estimate it on average. It is an exciting and active field of mathematics that has connections with Analysis, Geometry, and Number Theory.

At the beginning of the course we concentrate on presenting many fundamental examples of dynamical systems (such as Circle Rotations, the Baker Map, the Continued Fraction Map, and others). Motivated by theses examples, we introduce some of the important notions that one is interested in studying. Then in the second part of the course we will formalise these concepts and cover the basic definitions and some of the fundamental results in Topological Dynamics, Symbolic Dynamics, and Ergodic Theory. During the course we also discuss applications to other areas of mathematics and to concrete problems such as, for instance, Internet search engines.

Course Assessment: The course will be assessed by a standard 2.5 hour written examination (90%) together with assessed weekly homework (10%), which will consist of 10 exercise sheets. Only the best seven out of ten will count towards your assessed homework average.

Syllabus:

- *Basic notions:* dynamical system, orbits, fixed points and fundamental questions;
- *Basic examples of dynamical systems:* circle rotations; the doubling map and expanding maps of the circle; the shift map; the bakers map; the CAT map and hyperbolic toral automorphisms; the Gauss transformation and Continued Fractions;
- *Topological Dynamics:* basic metric spaces notions; minimality; topological conjugacy; topological mixing; topological entropy; topological entropy of toral automorphisms;
- *Symbolic Dynamics:* Shifts and subshifts spaces; topological Markov chains and their topological dynamical properties; symbolic coding; coding of the CAT map;
- *Ergodic Theory:* basic measure theory notions; invariant measures; Poincare' Recurrence; ergodicity; mixing; the Birkhoff Ergodic Theorem; Markov measures; Perron-Frobenius theorem, the ergodic theorem for Markov chains and applications to Internet Search.

References:

1. L. Barreira and C. Valls, Dynamical Systems: An Introduction, Springer, 2012.

2. M. Brin and G. Stuck, Introduction to Dynamical Systems, Cambridge University Press, 2015.

3. B. Hasselblatt and A. Katok, A First Course in Dynamics: with a Panorama of Recent Developments, Cambridge University Press, 2003.

4. M. Pollicott and M. Yuri, Dynamical Systems and Ergodic Theory, Cambridge University Press, 1998.