Functional Analysis: Presentations
(students of level M and higher)

1. *Classical trigonometric series* (Ch. 5 of [4], or §4.23-4.26 of [3]).

   Discuss completeness of the trigonometric set $\{e^{inx}\}$ and density of trigonometric polynomials.

2. *Pointwise convergence of Fourier series* (§4.7-5 of [1], or §5.11-5.12 or [3]).

   Give example of continuous function with divergent Fourier series.

3. *Uniformly convex Banach spaces and their properties* (§5.2 of [2], or §2.5-4 and §6.2 of [1]).

   Discuss non-compactness of unit ball in infinite-dimensional Banach spaces and the closed point property.

4. *Hilbert–Schmidt operators* (§8.1 of [4] or §3.1 of [5]).

   Discuss Hilbert-Schmidt operators and compactness of integral operators.

5. *Minkowski norm and separation of convex sets* (§3.2 of [2]).

   Discuss the Minkowski functional and the separation properties of convex sets.

6. *Numerical integration and weak* convergence* (§4.11 of [1]).

7. *Reflexive spaces* (§4.6 of [1]).

8. *Summability of sequences* (§4.10 of [1]).

9. *Invariant extensions of linear functionals* (§3.3 of [2]).

10. *Positive linear functionals and existence of finitely additive invariant measure on the circle* (§4.1 and §4.3 of [2]).

11. *Use of complex analysis in spectral theory* (§7.5 of [1])

    This topic requires some knowledge of complex analysis.

References

[1] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 1989.

[2] D. Lax, *Functional Analysis*, Wiley lnterscience, New York, 2002.

[3] W. Rudin, *Real and complex analysis*, John Wiley & Sons Inc., 2002.

[4] N. Young, *An Introduction to Hilbert Space*, Cambridge University Press, 1988.

[5] R. Zimmer, *Essential Results in Functional Analysis*, University of Chicago Press, 1990.